یازدهم دهم C $\quad$ 19,75 $\quad$ فاطمه ثابت - ۹

Date $\quad$ No

1- $f(x) = \begin{cases} \cot \frac{\pi x}{4} & ; x \le 1 \\ \sqrt{x^2+1} & ; x > 1 \end{cases}$ $\qquad fof\left(\frac{2}{3}\right) = f\left(\cot \frac{\pi}{6}\right) = \sqrt{(\sqrt{3})^2+1} = 2$ $\qquad$ (2)

2- الف) $f\left(\frac{1}{x}\right) = \sqrt{\frac{2x-1}{x^2}} \qquad g(x) = 2\cos^2 x \qquad f\left(g\left(\frac{\pi}{3}\right)\right) = f\left(\underbrace{2\cos^2\frac{\pi}{3}}_{\frac{1}{4}\times 2}\right) = f\left(\frac{1}{2}\right) = 0$

$f\left(\frac{1}{2}\right) = f\left(\frac{1}{x}\right) \to x = 2 \qquad f\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$

ب) $f(x) = [x] \qquad g(x) = \frac{x}{1-x} \qquad f(g(\sqrt{2})) = f\left(\frac{\sqrt{2}}{1-\sqrt{2}}\right) = \left[\frac{\sqrt{2}}{1-\sqrt{2}}\right]$ $\qquad$ (1,5)

$\to \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = -\underbrace{\sqrt{2}}_{1,\dots} - 2 = -3,\dots \Rightarrow [-3,\dots] = -4 \Rightarrow fog(\sqrt{2}) = -4$

3- $f(x) = \sin x \qquad g\left(f\left(\frac{\pi}{4}\right)\right) = g\left(\sin\frac{\pi}{4}\right) = g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$

$g(x) = x\sqrt{1-x^2}$ $\qquad$ (2)

4- $f(x) = \{(4,5)(6,5)(8,13)(10,12)\}$

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الف) $fog(x) = \{(4,5)(2,5)(6,12)(8,12)\}$ $\qquad$ (5)

ب) $gof(x) = \varnothing$

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د) $gog(x) = \{(4,8)(2,6)(6,10)\}$

5- $f = \{(2,1)(3,2)(4,5)(1,7)\} \qquad (4,2) \in fog \to f(g(4)) = 2 \to f(3) = 2 \Rightarrow g(4) = 3$

$g = \{(1,2)(3,1)(a,3)(b,1)\} \qquad (4,1) \in gof \to g(f(4)) = 1 \to g(5) = 1 \Rightarrow b = 5$

$\Rightarrow a = 4$

$\to \left.\begin{matrix} b = 5 \\ a = 4 \end{matrix}\right\} \Rightarrow (a,b) = (4,5) \in f(x)$ $\qquad$ (5)

6- $f(f(x)) = 4x+3 \qquad g(2x+3) = 3x-2$

$\to f(x) = ax+b \to f(ax+b) = 4x+3 \Rightarrow a(ax+b)+b = 4x+3 \to a^2x+ab+b = 4x+3 \to a = \pm 2,$

$\left\{\begin{matrix} a=2 \to 2b+b=3 \to b=1 \to f(x) = 2x+1 \\ a=-2 \to -2b+b=3 \to b=-3 \to f(x) = -2x-3 \end{matrix}\right\} f(-1) = -1$ $\qquad$ (5)

$g(x) = cx+d \to g(2x+3) = c(2x+3)+d = 2cx+3c+d = 3x-2 \Rightarrow c = \frac{3}{2}, d = -\frac{13}{2}$

$\to g(x) = \frac{3}{2}x - \frac{13}{2}$

$gof(-1) = g(f(-1)) = g(-1) = -8$

7- $f(x)=\sqrt{x+|x|}$ $\quad g(x)=\dfrac{1}{x^2-4x}$ ۵

$\hookrightarrow \underbrace{x+|x|}_{+}\geq 0 \Rightarrow x\geq 0$

$D_{gof}=\{x\in D_f : f(x)\in D_g\} \longrightarrow D_{gof}=\{x\geq 0 : \underbrace{\sqrt{x+|x|}\neq \{0,4\}}\} \Rightarrow D_{gof}=(0,+\infty)-\{8\}$

$x\neq 0 \,,\, x+|x|\neq 16 \Rightarrow x\neq 8$

8- $f(x)=\sqrt{1-x^2}$ $\quad g(x)=\sqrt{x}$ $\quad (f+g)of \Rightarrow f+g=\sqrt{x}+\sqrt{1-x^2}$ ۲

$\longrightarrow (f+g)(f(x))=(f+g)(\sqrt{1-x^2})=\sqrt[4]{1-x^2}+\sqrt{1-1+x^2}=\sqrt[4]{1-x^2}+|x| \rightarrow 1-x^2\geq 0 \rightarrow 1\geq x^2 \Rightarrow 1\geq x\geq -1$

$\Rightarrow D_{(f+g)of}=[-1,1]$

9- الف) $f\left(\dfrac{3x+1}{x-2}\right)=4x+5 \longrightarrow t=\dfrac{3x+1}{x-2} \Rightarrow x=\dfrac{2t+1}{t-3} \longrightarrow f(t)=4\left(\dfrac{2t+1}{t-3}\right)+5 \Rightarrow f(x)=\dfrac{13x-11}{x-3}$

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ب) $f\left(x+\dfrac{1}{x}\right)=x^3+\dfrac{1}{x^3} \longrightarrow t=x+\dfrac{1}{x} \longrightarrow \left(x+\dfrac{1}{x}\right)^3-3\left(x+\dfrac{1}{x}\right)=x^3+\dfrac{1}{x^3} \Rightarrow f(t)=t^3-3t \Rightarrow f(x)=x^3-3x$

10- $f(x)=x\sqrt{x}$ $\quad gof(x)$ $\quad g(1)=g(2\sqrt{2})=0$ ۵

$gof(x)=g(f(x))=g(x\sqrt{x})=0 \longrightarrow g(1)=0 \rightarrow 1=x\sqrt{x} \Rightarrow x=1$

$\searrow g(2\sqrt{2})=0 \rightarrow 2\sqrt{2}=x\sqrt{x} \Rightarrow x=2$

$\Rightarrow (x_2-x_1)=1$