

$$f(x) = \begin{cases} \cot \frac{\pi x}{4}, & x \leq 1 \\ \sqrt{x^2+1}, & x > 1 \end{cases}$$

$$f\left(\frac{1}{4}\right) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{4} \rightarrow \cot \frac{\pi}{4} = \frac{\sqrt{3}}{\frac{1}{4}} = \sqrt{3}$$

$$f \circ f\left(\frac{1}{4}\right) = ?$$

سوال (۱)

$$f(\sqrt{3}) = \sqrt{(\sqrt{3})^2+1} = \sqrt{4} = 2$$

$$f\left(\frac{1}{n}\right) = \sqrt{\frac{1}{n^2}-1}, \quad g(n) = 2 \cos^2 n, \quad f \circ g\left(\frac{1}{4}\right) = ?$$

سوال (۲)

$$\sqrt{\frac{1}{n^2}-1} = \sqrt{\frac{1-n^2}{n^2}} = \frac{\sqrt{1-n^2}}{n} \Rightarrow f \circ g\left(\frac{1}{4}\right) = \sqrt{2\left(\frac{1}{4}\right) - \left(\frac{1}{4}\right)^2} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

$$f(n) = [n], \quad g(n) = \frac{n}{1-n}, \quad f \circ g(\sqrt{2}) = ?$$

$$f \circ g\left[\frac{n}{1-n}\right] \xrightarrow{n=\sqrt{2}} \left[\frac{\sqrt{2}}{1-\sqrt{2}}\right] = [-1.5] = -2$$

$$f(n) = \sin n, \quad g(n) = n\sqrt{1-n^2}, \quad g \circ f\left(\frac{\pi}{4}\right) = ?$$

سوال (۳)

$$f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}, \quad g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \left(\sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2}\right) = \frac{\sqrt{2}}{2} \left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2}$$

$$f(n) = \{(4, 5), (4, 5), (1, 12), (8, 12)\}, \quad g(n) = \{(4, 4), (2, 4), (4, 8), (1, 10)\}$$

سوال (۴)

$$f \circ g(n) = \{(4, 5), (2, 5), (4, 12), (8, 12)\}$$

$$g \circ f(n) = \emptyset$$

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$$g \circ g(n) = \{(4, 8), (2, 4), (4, 10)\}$$

$$f = \{(2, 1), (3, 2), (4, 5), (1, 7)\}, \quad g = \{(1, 2), (3, 1), (a, 3), (b, 1)\}, \quad (4, 2) \in f \circ g$$

سوال (۵)

$$(4, 2) \in f \circ g \rightarrow f(g(4)) = 2 \rightarrow f(3) = 2 \rightarrow g(4) = 3 \rightarrow a = 4$$

$$(4, 1) \in g \circ f$$

$$(a, b) = ?$$

$$(4, 1) \in g \circ f \rightarrow g(f(4)) = 1 \rightarrow f(5) = 1 \rightarrow g(5) = 1 \rightarrow b = 5$$

$$(a, b) = (4, 5)$$

$$f(f(n)) = 4n+3, \quad g(2n+3) = 3n-2, \quad g \circ f(-1) = ?$$

سوال (۶)

$$f \circ f = 4n+3$$

$$g(n) = \frac{3}{4}n - \frac{13}{4}$$

$$g \circ f(-1) = -8$$

$$a(4n+3)+b \rightarrow a^2n+(ab+b)=4n+3 \rightarrow a=4, \quad b=1 \rightarrow 4n+1$$

$$g \circ f(-1) \rightarrow \begin{cases} \frac{3}{4}(4n+1) - \frac{13}{4} \rightarrow -8 \\ \frac{3}{4}(-4n-3) - \frac{13}{4} \rightarrow -8 \end{cases}$$

$$f(n) = \sqrt{n+|n|}, \quad g(n) = \frac{1}{n^2-4n}, \quad D_{g \circ f} = ?$$

سوال (۷)

$$g \circ f: \frac{1}{(\sqrt{n+|n|})^2 - 4(\sqrt{n+|n|})} = \frac{1}{(n+|n|) - 4\sqrt{n+|n|}} \Rightarrow D_{g \circ f} = (0, +\infty) - \{4\}$$

سوال ۸

$f(x) = \sqrt{1-x^2}$, $g(x) = \sqrt{x}$, $(f+g) \circ f = ?$

$f+g \rightarrow \sqrt{x} + \sqrt{1-x^2}$

$(f+g) \circ f \rightarrow \sqrt{x^2} + \sqrt{1-x^2} \rightarrow 1-x^2 > 0 \rightarrow 1 > x^2 > -1$

$\Rightarrow D = [-1, 1]$

الف $f\left(\frac{3x+1}{x-2}\right) = 3x+5$

$\frac{3x+1}{x-2} = t \rightarrow t(x-2) = 3x+1 \rightarrow tx-2t = 3x+1 \rightarrow$

$tx-3x = 1+2t \rightarrow x(t-3) = 1+2t \rightarrow x = \frac{1+2t}{t-3}$

$f\left(\frac{3x+1}{x-2}\right) = f\left(\frac{1+2t}{t-3}\right) + 5 \rightarrow f(t) = \frac{13t-11}{t-3}$

$\Rightarrow f(x) = \frac{13x-11}{x-3}$

سوال ۹

ب $f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3}$

$t = x + \frac{1}{x} \rightarrow t^3 = \left(x + \frac{1}{x}\right)^3 \Rightarrow t^3 = x^3 + \frac{1}{x^3} + \frac{3}{x} + 3x \rightarrow t^3 = \left(x^3 + \frac{1}{x^3}\right) + 3\left(x + \frac{1}{x}\right)$

$\Rightarrow \left(x^3 + \frac{1}{x^3}\right) = t^3 - 3t$

$\Rightarrow f(x) = x^3 - 3x$

$f(x) = x\sqrt{x}$

$x=0 \rightarrow y=0$

نقاطی که تابع و محور x ها را قطع می کند : $(2\sqrt{2}, 0)$, $(1, 0)$

سوال ۱۰

نقاطی که تابع و محور x ها را قطع می کند : $(2\sqrt{2}, 0)$, $(1, 0)$

$\Rightarrow \begin{cases} f(x) = 1 \rightarrow x\sqrt{x} = 1 \rightarrow x = 1 \\ f(x) = 2\sqrt{2} \rightarrow x\sqrt{x} = 2\sqrt{2} \rightarrow x = 2 \end{cases}$

اختلاف طول نقاط $= 2-1 = 1$