۱- $\frac{2}{3}<1 \to f(\frac{2}{3}) = \cot\frac{2\pi}{3\times 2} = \cot\frac{\pi}{6} = \sqrt{3}$ $\quad f(\sqrt{3}) = \sqrt{3+1} = \sqrt{4} = \boxed{2}$

$\sqrt{3}>1$

۲- الف) $g(\frac{\pi}{3}) = 2\cos^2\frac{\pi}{3} = 2\times(\frac{1}{2})^2 = 2\times\frac{1}{4} = \frac{1}{2}$ $\quad f(\frac{1}{2}) = \sqrt{\frac{(2\times 2)-1}{4}} = \sqrt{\frac{3}{4}} = \boxed{\frac{\sqrt{3}}{2}}$

ب) $g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}(1+\sqrt{2})}{-1} = \boxed{-2-\sqrt{2}}$

$f(-2-\sqrt{2}) = [-2-\sqrt{2}] = -2 + [-\sqrt{2}] = -2-2 = \boxed{-4}$

$-2<-\sqrt{2}<-1$

۳- $f(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \to g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{2}} = \frac{\cancel{\sqrt{2}}}{2}\times\frac{1}{\cancel{\sqrt{2}}} = \boxed{\frac{1}{2}}$

۴- الف) $g(4) = 6 \to f(6) = 5$

$g(2) = 4 \to f(4) = 5$

$g(6) = 8 \to f(8) = 12$

$g(8) = 10 \to f(10) = 10$

$\to fog(x) = \{(2,5)(4,5)(6,12)(8,12)\}$

ب) $\varnothing \to \{5,12\} \notin D_g$

ج) $\varnothing \to \{5,12\} \notin D_g$

د) $g(6) = 8 \to g(8) = 10$

$g(2) = 4 \to g(4) = 6$

$g(4) = 6 \to g(6) = 8$

$\to gog(x) = \{(6,10)(2,6)(4,8)\}$

۵- $f(3) = 2 \to g(a) = 3 \Rightarrow a = 4$

$f(4) = 5 \to g(b) = 1 \to b = 5$

$\to (a,b) = (4,5)$

$f(f(-1)) = f(-1)+3 = -1 \quad f(x) = -1 \quad 4x+3 = -1 \to 4x = -4 \to \boxed{x = -1}$ ۶

$\Rightarrow f(-1) = -1 \to f(-1) = \boxed{-1}$

$2x+3 = -1 \to 2x = -4 \to \boxed{x = -2} \to g(-1) = 3(-2)-2 = \boxed{-8}$

$D_{gof} = \{ x \in D_f \, , \, f(x) \in D_g \}$ ۷
$x \in R$ (1)

$D_f \Rightarrow x + |x| \geq 0 \to x \in R$

$g(x) = \dfrac{1}{x(x-4)} \to D_g = R - \{0, 4\} \to \begin{cases} \sqrt{x+|x|} \neq 0 \to x = (0, +\infty) \text{ (2)} \\ \sqrt{x+|x|} \neq 4 \xrightarrow{\text{به توان ۲}} x+|x| \neq 16 \text{ (3)} \end{cases}$

$D_{gof} = 1 \cap 2 \cap 3 = (0, +\infty) - \{8\} \qquad \xrightarrow{x>0} 2x \neq 16 \to x \neq 8$

$D_{(f+g)of} = \{ x \in D_f \, , \, f(x) \in D_{(g+f)} \}$ ۸

$f(x) = \sqrt{1-x^2} \to 1-x^2 = 0 \to x = \pm 1 \qquad \begin{array}{ccccc} & -1 & & 1 & \\ - & & + & & - \end{array} \to D_f = [-1, 1]$ (1)

$D(f+g) = D_f \cap D_g = [-1, 1] \cap [0, +\infty) = [0, 1]$

$\to 0 \leq \sqrt{1-x^2} \leq 1 \xrightarrow{\text{به توان ۲}} 0 \leq 1-x^2 \leq 1 \longrightarrow x^2 \geq 0 \to D_{(g+f)} = \mathbb{R}$ (2)

$D_{(f+g)of} = 1 \cap 2 = [-1, 1]$

الف) $\dfrac{3x+1}{x-2} = t \to x = -\dfrac{-2t-1}{t-3} = \dfrac{2t+1}{t-3}$ ۹

$f(t) = \dfrac{8t+4}{t-3} + 5 = \dfrac{8t+4+5t-15}{t-3} = \dfrac{13t-11}{t-3} \longrightarrow \boxed{f(x) = \dfrac{13x-11}{x-3}}$

ب) $x + \dfrac{1}{x} = t \to t^3 = x^3 + \dfrac{1}{x^3} + \underbrace{\left(x \times \dfrac{1}{x}\right)}_{1} \underbrace{\left(x + \dfrac{1}{x}\right)}_{t} \to x^3 + \dfrac{1}{x^3} = t^3 - t$

$f(t) = t^3 - t \to \boxed{f(x) = x^3 - x}$

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$g(f(x))' = 0 \longrightarrow f(x) = ۱ \rightarrow x\sqrt{x} = ۱ \rightarrow x = ۱$ ۱۰

$\longrightarrow f(x) = ۲\sqrt{۲} \longrightarrow x\sqrt{x} = ۲\sqrt{۲} \longrightarrow x = ۲$

$\Big\} \longrightarrow ۲ - ۱ = \boxed{۱}$