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الف ۹

از دهم دفتر C

موسی و صفا

$$f(n) = \begin{cases} \cos \frac{\pi n}{K} & n \leq 1 \\ \sqrt{n^2 + 1} & n > 1 \end{cases}$$

①

②

$$\log\left(\frac{r}{K}\right) \rightarrow f\left(f\left(\frac{r}{K}\right)\right)$$

$$f\left(\frac{r}{K}\right) = \cos \frac{\pi}{4} = \sqrt{2} \rightarrow f(\sqrt{2}) = \sqrt{(\sqrt{2})^2 + 1} = 2$$

$$\text{الف) } f\left(\frac{1}{n}\right) = \sqrt{\frac{r_{n-1}}{n^r}} \quad g(n) = r \cos^n n \quad \text{②}$$

$$\log\left(\frac{\pi}{K}\right) \rightarrow g\left(\frac{\pi}{K}\right) = r_n \frac{1}{K} = \frac{1}{r}$$

$$f\left(\frac{1}{n}\right) = f\left(\frac{1}{r}\right) \rightarrow n \geq r \quad f(r) = \sqrt{\frac{r-1}{K}} = \frac{\sqrt{r}}{r}$$

$$\Rightarrow f(n) = [n] \quad g(n) = \frac{n}{1-n}$$

$$\log(\sqrt{2}) \rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{-1}$$

$$= -\sqrt{2}-2$$

$$f(-\sqrt{2}-2) = [-\sqrt{2}-2] = -K$$

$$f(n) = \sin n$$

$$g(n) = n\sqrt{1-n^2} \quad \text{god}\left(\frac{\pi}{4}\right) \rightarrow f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4}$$

$$g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \frac{2}{4}} = \frac{\sqrt{2}}{2} \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{1}{2}$$

$$f(n) = \{(4, 5), (9, 5), (1, 12), (10, 12)\}$$

$$g(n) = \{(4, 9), (2, 4), (9, 1), (1, 10)\}$$

$$\text{Log} = \{(4, 5), (2, 5), (9, 12), (1, 12)\}$$

$$\text{god}(n) = \emptyset \quad \text{Log}(n) = \emptyset$$

$$\text{god}(n) = \{(4, 1), (2, 4), (4, 10)\}$$

$$f(n) = \{(4, 1), (3, 2), (4, 5), (1, 7)\}$$

$$g(n) = \{(1, 2), (3, 1), (a, 3), (b, 1)\}$$

$$(4, 2) \in \text{Log} \quad g(n) = t \rightarrow f(t) = 2 \rightarrow t = 4$$

$$n = a \rightarrow a = 4$$

$$(4, 1) \in \text{god} \quad g(f(n)) = 1 \quad f(n) = b$$

$$(4, 5)$$

$$b = 5$$

$$(a, b) = (4, 5)$$

ولادت حضرت ابوالفضل العباس (ع) (۲۶ هجری) و روز جانباز

$$f(n) = n^2$$

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$$g(n) = n^2$$

$$f(n) \rightarrow a(n+b) + b = n^2$$

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$$a^2n + ab + b = n^2$$

$$a = \pm 1 \rightarrow \begin{cases} a = 1 \rightarrow b = 1 \\ a = -1 \rightarrow b = -1 \end{cases}$$

$$g(n) = cn + d \rightarrow c(n^2) + d = n^2$$

$$n = n^2$$

$$cn + d = n^2$$

$$c = \frac{1}{2}, d = -\frac{1}{2}$$

$$f(n) = n^2 \rightarrow g(f(-1)) = -1$$

$$f(n) = -n^2 \rightarrow g(f(-1)) = -1$$

$$f(n) = \sqrt{n+1} \quad f(n) = \begin{cases} \sqrt{n} & n \geq 0 \\ 0 & n < 0 \end{cases}$$

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$$g(n) = \frac{1}{n^2 - n}$$

$$D_f = \mathbb{R}$$

$$D_g = \mathbb{R} - \{0, 1\}$$

$$\sqrt{n+1} \neq \{0, 1\} \rightarrow n \geq 0$$

$$L \rightarrow n \neq 1$$

$$D_{g \circ f} = (0, +\infty) - \{1\}$$

$$f(x) = \sqrt{1-x^2} \quad g(x) = \sqrt{x} \quad (1)$$

$$\{x \in D_f, f(x) \in D_{g \circ f}\} \quad D_f = [-1, 1] \quad D_{g \circ f} = [0, +\infty)$$

$$\sqrt{1-x^2} \in [0, 1] \rightarrow 0 \leq \sqrt{1-x^2} \leq 1 \rightarrow 0 \leq 1-x^2 \leq 1$$

$$-1 \leq -x^2 \leq 0 \rightarrow -1 \leq x \leq 1 \rightarrow [-1, 1] \quad (5)$$

$$D_f \cap D_{g \circ f} = [0, 1]$$

$$f\left(\frac{n+1}{n-1}\right) = F_{n+1}$$

$$\frac{n+1}{n-1} = t \quad f(t) = F\left(\frac{t+1}{t-1}\right) + 1$$

$$n = \frac{t+1}{t-1} \rightarrow f(n) = \frac{F_{n-1}}{n-1} \quad (5)$$

$$\Rightarrow f\left(n + \frac{1}{n}\right) = n^k + \frac{1}{n^k}$$

$$n + \frac{1}{n} = t \quad t^k = n^k + \frac{1}{n^k} + k\left(n + \frac{1}{n}\right)$$

$$t^k = n^k + \frac{1}{n^k} + kt \rightarrow t^k - kt = \frac{1}{n^k} + n^k$$

$$f(t) = t^k - kt \rightarrow f(n) = n^k - kn$$

$$f(n) = n\sqrt{n} \quad n \geq 0$$

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$$g(f(n)) = 0 \rightarrow g(n/2) \quad n_1 = 1 \quad n_2 = \sqrt{n}$$

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$$f(n) \begin{cases} n \geq 1 \rightarrow n\sqrt{n+1} \rightarrow n'_1 = 1 \\ n_2 = \sqrt{n+1} \rightarrow n\sqrt{n+2} \rightarrow n'_2 = 2 \end{cases} \quad n'_1 - n'_2 = 1$$

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