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تکلا محسنی بنام خدا

۱- $fof(\frac{2}{3}) = \boxed{2}$

$f(\frac{2}{3}) = Cot\frac{2}{3}\times\frac{\pi}{4} = Cot\frac{\pi}{6} = \sqrt{3}$ $\quad f(\sqrt{3}) = \sqrt{3+1} = 2$

۲-

الف) $fog(\frac{\pi}{3}) = \boxed{\frac{\sqrt{3}}{2}}$

$g(\frac{\pi}{3}) = 2Cos^2\frac{\pi}{3} = \frac{1}{2}$ $\quad f(\frac{1}{2}) = \sqrt{\frac{4-1}{4}} = \frac{\sqrt{3}}{2}$

ب) $fog(\sqrt{2}) = \boxed{-4}$

$g(\sqrt{2}) = \frac{\sqrt{2}\times(1+\sqrt{2})}{1-\sqrt{2}\times(1+\sqrt{2})} = \frac{2+\sqrt{2}}{-1} = -2-\sqrt{2}$ $\quad f(-2-\sqrt{2}) = [-2-\sqrt{2}] = [-3.4] = -4$

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۳- $gof(\frac{\pi}{4}) = \boxed{\frac{1}{2}}$

$f(\frac{\pi}{4}) = Sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{2}} = \frac{\sqrt{2}}{2}\times\frac{1}{\sqrt{2}} = \frac{1}{2}$

۴-

الف) $fog(x) = \{(4,5),(2,5),(6,12),(8,12)\}$

ب) $gof(x) = \varnothing$

ج) $fof(x) = \varnothing$

د) $gog(x) = \{(4,8),(2,6),(6,10)\}$

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۵_ $fog(4)=2 \longrightarrow f(g(4))=2 \longrightarrow a=4$

$g(f(4))=1 \longrightarrow g(5)=1 \longrightarrow b=5 \qquad (a,b) \longrightarrow (4,5)$

۶_ $f(x)=ax+b \rightarrow f(ax+b)=a^2x+ab+b=4x+3 \begin{cases} a=2 & b=1 \\ a=-2 & b=-3 \end{cases}$

$g(x)=a'x+b' \longrightarrow g(2x+3)=2a'x+3a'+b'=3x-2 \longrightarrow a'=\frac{3}{2} \quad b'=-\frac{13}{2}$

$f(-1)=-1\times 2+1=-1 \quad g(-1)=-1\times\frac{3}{2}-\frac{13}{2}=-8 \Rightarrow gof(-1)=-8$

۷_ $f(x)=\sqrt{x+|x|} \qquad g(x)=\frac{1}{x^2-4x}$

$D_{gof}=\{x\in D_f \,,\, f\in D_g\}$

$D_f=$ I

$0 \quad \sqrt{2x}$

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$g(x)=\frac{1}{x(x-4)} \longrightarrow D_g=\mathbb{R}-\{0,4\}$

$f(x)=4 \longrightarrow \sqrt{2x}=4 \longrightarrow x=8$

$f(x)=0 \longrightarrow (-\infty,0]$ II

$\mathrm{I}\cap\mathrm{II} \Rightarrow D_{gof}=(0,+\infty)-\{8\}$

۸_ $D_{(f+g)of}=\{x\in D_f \,,\, f(x)\in D_{(f+g)(x)}\}$

$f=\sqrt{1-x^2} \longrightarrow D_f=[-1,1]$ I

$0\leq\sqrt{1-x^2}\leq 1 \longrightarrow 1-x^2\leq 1 \longrightarrow 0\leq x^2 \longrightarrow \mathbb{R}$ II

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$\xrightarrow{\mathrm{I}\cap\mathrm{II}} D_{(f+g)of}=[-1,1]$

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۹. الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5 \rightarrow \frac{3x+1}{x-2} = t \rightarrow x = \frac{2t+1}{t-3}$

$$4\left(\frac{2t+1}{t-3}\right)+5 = \frac{13t-11}{t-3} \Longrightarrow f(x) = \frac{13x-11}{x-3}$$

ب) $f\left(x+\frac{1}{x}\right) = x^3+\frac{1}{x^3} \longrightarrow x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$

$$\longrightarrow f(x) = x^3 - 3x$$

۱۰- $g(x) = a(x-1)(x-2\sqrt{2})+b$

$gof = a(x\sqrt{x}-1)(x\sqrt{x}-2\sqrt{2})+b \xrightarrow{\text{ریشه ها}} x = 2, 1 \longrightarrow 2-1 = 1$