

$$f(f(n)) = 2n+3 \quad g(f(n)) = n-1 \quad g \circ f(-1) = ? \rightarrow f(f(n)) = a(an+b) + b \Rightarrow an + ab + b = 2n+3 \quad (2 \text{ سوال})$$

$$\Rightarrow a' = 2 \rightarrow a = 2 \rightarrow 2b + b = 2b + 3 \rightarrow b = 1 \xrightarrow{D} f(n) = 2n+1$$

$$L: a = 2 \rightarrow 2b + b = 3 \rightarrow b = 1 \xrightarrow{D} f(n) = 2n+1$$

$$2n+3 = f \Rightarrow 2n+3 = 2 \Rightarrow n = \frac{3-2}{2} \Rightarrow g(f) = 2n \frac{f-3}{2} = 2 \frac{f-3}{2} = \frac{f-3}{1} = \frac{f}{1} - \frac{3}{1}$$

$$\textcircled{1} \rightarrow g \circ f(-1) = g(f(-1)) = g(-1) = \frac{f}{1}(-1) - \frac{3}{1} = \frac{-1}{1} - \frac{3}{1} = -1 - 3 = -4 \quad f(-1) = 2(-1) + 1 = -1$$

$$\textcircled{2} \rightarrow g \circ f(-1) = g(f(-1)) = g(-1) = -4 \quad f(-1) = 2(-1) + 1 = -1$$

$$g \circ f(n) \begin{cases} n \in D_f \rightarrow n+|n| \geq 0 \xrightarrow{n \geq 0} n \geq 0 \text{ قابل قبول} \\ f \in D_g \end{cases}$$

$$\begin{cases} n > 0 \rightarrow 0 < 0 \text{ " " } D_f = \mathbb{R} \\ n < 0 \rightarrow -n+n \geq 0 \end{cases}$$

$$D_{g \circ f} = (0, +\infty) - \{1\} \cup (0, 1) \cup (1, +\infty)$$

$$n^2 - 2n \neq 0$$

$$n \neq 0, n \neq 2$$

$$\sqrt{n+|n|} \neq 0 \Rightarrow n+|n| \neq 0 \Rightarrow n > 0$$

$$\sqrt{n+|n|} \neq 1 \Rightarrow n+|n| \neq 1 \Rightarrow n \neq 1$$

$$(f+g) \circ f = f(f(n)) + g(f(n)) = \sqrt{1-(\sqrt{1-n^2})^2} + \sqrt{1-n^2} = 1-n^2 \geq 0 \Rightarrow n^2 \leq 1 \Rightarrow -1 \leq n \leq 1 \quad (3 \text{ سوال})$$

$$1 - (\sqrt{1-n^2})^2 \geq 0 \Rightarrow (\sqrt{1-n^2})^2 \leq 1 \Rightarrow -1 \leq \sqrt{1-n^2} \leq 1 \Rightarrow 1-n^2 \leq 1 \Rightarrow n^2 \geq 0 \quad D_{(f+g) \circ f} = [-1, 1]$$

$$\text{ب) } f\left(\frac{3n+1}{n-2}\right) \in \mathbb{N} \Rightarrow \frac{3n+1}{n-2} \in \mathbb{Z} \Rightarrow \frac{3n+1}{n-2} = 3n+1 \quad (3-2)n = 2n+1 \Rightarrow n = \frac{2n+1}{3-2}$$

$$f(t) = 3 \times \frac{2t+1}{t-2} + 2 = \frac{6t+3}{t-2} + 2 = \frac{6t+3+2t-4}{t-2} = \frac{8t-1}{t-2} \quad f(n) = \frac{8n-1}{n-2}$$

$$\rightarrow f\left(n+\frac{1}{n}\right) = n^3 + \frac{1}{n^3} \Rightarrow f(t) = t^3 - 2t \Rightarrow f(n) = n^3 - 2n$$

$$\left(n+\frac{1}{n}\right)^3 = n^3 + \frac{1}{n^3} + 3 \times n \times \frac{1}{n} \left(n+\frac{1}{n}\right) \Rightarrow n^3 + \frac{1}{n^3} + 3 \left(n^2 + 1\right) = \left(n^3 - 2n\right)$$

$$g(1) = 0, g(\sqrt{2}) = 0 \quad f(n) = n\sqrt{n}$$

$$g \circ f(n) = g(f(n)) = 0 \rightarrow f(n) = 1 \Rightarrow n = 1$$

$$\rightarrow f(n) = \sqrt{2} \Rightarrow n = 2$$

$$n-1 \geq 1$$

(1. سوال)

سوال (۱)

$$f(n) = \begin{cases} \cot \frac{\pi n}{e} & : n \leq 1 \\ \sqrt{n^2 - 1} & : n > 1 \end{cases}$$

$$f(f(\frac{e}{e})) \Rightarrow f(\frac{e}{e}) : \cot \frac{e\pi}{ee} = \cot \frac{\pi}{e} = \sqrt{e}$$

$$f(\sqrt{e}) : \sqrt{(\sqrt{e})^2 - 1} = \sqrt{e-1}$$

سوال (۲)

$$f(\frac{1}{n}) = \sqrt{\frac{e n - 1}{n^2}} \text{ و } g(n) = 2 \cos^2 n, f \circ g(\frac{\pi}{e})$$

$$f(\frac{1}{n}) = \sqrt{\frac{e n - 1}{n^2}} \rightarrow \frac{1}{n} = f : \frac{e n - 1}{n^2} = \frac{e n}{n^2} - \frac{1}{n^2} = \frac{e}{n} - \frac{1}{n^2} = 2f - f^2$$

$$f(f) = \sqrt{f - f^2} \Rightarrow f(n) = \sqrt{e n - n^2}$$

$$f(g(\frac{\pi}{e})) \Rightarrow g(n) = 2 \cos^2 n \xrightarrow{n = \frac{\pi}{e}} 2 \cos^2 \frac{1}{e} = \frac{1}{e^2}$$

$$\Rightarrow f(\frac{1}{e^2}) = \sqrt{e \cdot \frac{1}{e^2} - (\frac{1}{e^2})^2} = \sqrt{1 - \frac{1}{e^2}} = \sqrt{\frac{e^2 - 1}{e^2}} = \frac{\sqrt{e^2 - 1}}{e}$$

$$f(n) = [n] \text{ و } g(n) = \frac{n}{1-n} \text{ و } f \circ g(\sqrt{2})$$

$$f(g(\sqrt{2})) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \approx \frac{1.414}{1-1.414} = \frac{1.414}{-0.414} = -3.414$$

$$f(-3.414) = [-3.414] = -3$$

$$f(n) = \sin n \text{ و } g(n) = n \sqrt{1-n^2} \text{ و } g \circ f(\frac{\pi}{e}) = g(f(\frac{\pi}{e})) = ?$$

$$f(\frac{\pi}{e}) = \sin(\frac{\pi}{e}) = \frac{\sqrt{e}}{e}$$

$$g(\frac{\sqrt{e}}{e}) = \frac{\sqrt{e}}{e} \sqrt{1 - (\frac{\sqrt{e}}{e})^2} = \frac{\sqrt{e}}{e} \sqrt{1 - \frac{1}{e}} = \frac{1}{\sqrt{e}}$$

سوال (۳)

$$f(n) = \{(e, \alpha), (e, \alpha), (e, \alpha), (e, \alpha)\} \text{ و } g(n) = \{(e, \alpha), (e, \alpha), (e, \alpha), (e, \alpha)\}$$

سوال (۴)

$$f \circ g(n) : f \xrightarrow{g(n)} \alpha \rightarrow \alpha$$

$$e \rightarrow e \rightarrow \alpha$$

$$e \rightarrow e \rightarrow \alpha$$

$$e \rightarrow e \rightarrow \alpha$$

$$g \circ f(n) : f \xrightarrow{f(n)} \alpha \rightarrow \alpha$$

$$e \rightarrow \alpha \rightarrow \alpha$$

$$e \rightarrow \alpha \rightarrow \alpha$$

$$e \rightarrow \alpha \rightarrow \alpha$$

$$f \circ g(n) = \{(e, \alpha), (e, \alpha), (e, \alpha), (e, \alpha)\}$$

$$g \circ f(n) = \emptyset$$

$$f \circ f(n) : f \xrightarrow{f(n)} \alpha \rightarrow \alpha$$

$$e \rightarrow \alpha \rightarrow \alpha$$

$$e \rightarrow \alpha \rightarrow \alpha$$

$$e \rightarrow \alpha \rightarrow \alpha$$

$$g \circ g(n) : g \xrightarrow{g(n)} \alpha \rightarrow \alpha$$

$$e \rightarrow \alpha \rightarrow \alpha$$

$$e \rightarrow \alpha \rightarrow \alpha$$

$$e \rightarrow \alpha \rightarrow \alpha$$

$$f \circ f(n) = \emptyset$$

$$g \circ g(n) = \{(e, \alpha), (e, \alpha), (e, \alpha)\}$$

سوال (۵)

$$f(g(e)) = 2 \Rightarrow \boxed{2} \xrightarrow{g(n)} 3 \xrightarrow{f(n)} 2 \Rightarrow \boxed{2} = a = 2$$

$$(a, b) \Rightarrow (e, \alpha)$$

$$g(f(e)) = 1 \Rightarrow e \xrightarrow{f(n)} \alpha \xrightarrow{g(n)} 1 \Rightarrow (e, \alpha) \rightarrow (b, \alpha) \rightarrow b = e$$