سوال ۶)

$f(f(x))=4x+3 \quad g(2x+3)=3x-2 \quad gof(-1)=? \longrightarrow f(f(x))=a(ax+b)+b=a^2x+ab+b=4x+3$

$f(x)=ax+b$

$\Rightarrow a^2=4 \to a=2 \to 2b+b=3b=3 \to b=1 \xrightarrow{①} f(x)=2x+1$

$\quad \to a=-2 \to -2b+b=-b=3 \to b=-3 \xrightarrow{②} f(x)=-2x-3$

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$2x+3=t \Rightarrow 2x=t-3 \Rightarrow x=\frac{t-3}{2} \to g(t)=3\times\frac{t-3}{2}-2=\frac{3t-9-4}{2}=\frac{3}{2}t-\frac{13}{2}$

$① \to gof(-1)=g(f(-1))=g(-1)=\frac{3}{2}\times(-1)-\frac{13}{2}=\frac{-3}{2}-\frac{13}{2}=-8 \qquad f(-1)=-2+1=-1$

$② \to gof(-1)=g(f(-1))=g(-1)=\boxed{-8} \qquad f(-1)=-2\times(-1)-3=-1$

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سوال ۷)

$gof(x)\begin{cases} x\in D_f \to x+|x|\geq 0 \xrightarrow{x>0} 2x\geq 0 \text{ همواره برقرار} \\ f\in D_g \end{cases}$

$\quad x=0 \to 0=0 \quad$ " "

$\quad x<0 \to -x+x\geq 0 \quad$ " "

$D_f=\mathbb{R}$

۲

$x^2-4x\neq 0$

$x\neq 0 , x\neq 4$

$\sqrt{x+|x|}\neq 0 \Rightarrow x+|x|\neq 0 \Rightarrow x>0$

$\sqrt{x+|x|}\neq 4 \Rightarrow x+|x|\neq 16 \Rightarrow x\neq 8$

$D_{gof}=(0,+\infty)-\{8\}$ یا $(0,8)\cup(8,+\infty)$

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سوال ۸)

$(f+g)of=f(f(x))+g(f(x))=\sqrt{1-(\sqrt{1-x^2})^2}+\sqrt[4]{1-x^2} \quad 1-x^2\geq 0 \Rightarrow x^2\leq 1 \Rightarrow -1\leq x\leq 1$

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$1-(\sqrt{1-x^2})^2\geq 0 \Rightarrow (\sqrt{1-x^2})^2\leq 1 \Rightarrow \underbrace{-1\leq\sqrt{1-x^2}}_{\text{بدیهی}}\leq 1 \Rightarrow 1-x^2\leq 1 \Rightarrow \underbrace{x^2\geq 0}_{\text{بدیهی}} \qquad D_{(g+f)of}=[-1,1]$

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سوال ۹)

الف) $f\left(\frac{3x+1}{x-2}\right)=4x+5 \qquad \frac{3x+1}{x-2}=t \Rightarrow tx-2t=3x+1 \qquad (t-3)x=2t+1 \Rightarrow x=\frac{2t+1}{t-3}$

$f(t)=4\times\frac{2t+1}{t-3}+5=\frac{8t+4+5t-15}{t-3}=\frac{13t-11}{t-3} \qquad f(x)=\frac{13x-11}{x-3}$

ب) $f(x+\frac{1}{x})=x^3+\frac{1}{x^3} \Rightarrow f(t)=t^3-3t \Rightarrow f(x)=x^3-3x$

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$(x+\frac{1}{x})^3=x^3+\frac{1}{x^3}+3\times\underbrace{x\times\frac{1}{x}}_{1}\underbrace{(x+\frac{1}{x})}_{t} \Rightarrow x^3+\frac{1}{x^3}=\boxed{t^3-3t}=(x^3-3x)$

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سوال ۱۰)

$g(1)=0 , g(2\sqrt{2})=0 \qquad f(x)=x\sqrt{x}$

$\boxed{2-1=1}$

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$gof(x)=g(f(x))=0 \begin{cases} \to f(x)=1 \Rightarrow x=1 \\ \to f(x)=2\sqrt{2} \Rightarrow x=2 \end{cases}$

محور x ها را قطع می کند

Scanned with CamScanner

سوال ۱) ۵

$$f(x)\begin{cases}\cot\frac{\pi x}{4} & : x\le 1\\ \sqrt{x^2+1} & : x>1\end{cases}$$

$f(f(\frac{2}{3})) \Rightarrow f(\frac{2}{3}): \cot\frac{2\pi}{12} = \cot\frac{\pi}{6} = \sqrt{3}$

$f(\sqrt{3}): \sqrt{(\sqrt{3})^2+1} = \boxed{2}$

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سوال ۲) ۵

الف) $f(\frac{1}{x}) = \sqrt{\frac{2x-1}{x^2}}$ , $g(x) = 2\cos^2 x$ , $fog(\frac{\pi}{3})$

$f(\frac{1}{x}) = \sqrt{\frac{2x-1}{x^2}} \longrightarrow \frac{1}{x} = t : \frac{2x-1}{x^2} = \frac{2x}{x^2} - \frac{1}{x^2} = \frac{2}{x} - \frac{1}{x^2} = 2t - t^2$

$f(t) = \sqrt{2t-t^2} \Rightarrow f(x) = \sqrt{2x-x^2}$

$f(\underbrace{g(\frac{\pi}{3})}_{\frac{1}{2}}) \Rightarrow g(x) = 2\cos^2 x \xrightarrow[x=\frac{\pi}{3}]{} 2\times\frac{1}{4} = \frac{1}{2}$

$\Rightarrow f(\frac{1}{2}) = \sqrt{2\times\frac{1}{2} - (\frac{1}{2})^2} = \sqrt{1-\frac{1}{4}} = \sqrt{\frac{3}{4}} = \boxed{\frac{\sqrt{3}}{2}}$

ب) $f(x) = [x]$ , $g(x) = \frac{x}{1-x}$ , $fog(\sqrt{2})$

$f(\underbrace{g(\sqrt{2})}_{-2-\sqrt{2}}) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{-1} = -2-\sqrt{2}$

$f(-2-\sqrt{2}) \Rightarrow [\underbrace{-2-\sqrt{2}}_{-3,4\ldots}] = \boxed{-4}$

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سوال ۳) ۵

$f(x) = \sin x$ , $g(x) = x\sqrt{1-x^2}$ , $gof(\frac{\pi}{4}) = g(f(\frac{\pi}{4})) = ?$

$f(\frac{\pi}{4}) = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\underbrace{\sqrt{1-(\frac{\sqrt{2}}{2})^2}}_{\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{2}\times\frac{1}{\sqrt{2}} = \boxed{\frac{1}{2}}$

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سوال ۴) ۵

$f(x) = \{(4,5),(6,5),(8,12),(10,12)\}$ $g(x) = \{(4,6)(2,4)(6,8)(8,10)\}$

الف) $fog(x)$ : $4 \xrightarrow{g(x)} 6 \xrightarrow{f(x)} 5$

$2 \longrightarrow 4 \longrightarrow 5$

$6 \longrightarrow 8 \longrightarrow 12$

$8 \longrightarrow 10 \longrightarrow 12$

$\boxed{fog(x) = \{(4,5)(2,5)(6,12)(8,12)\}}$

ب) $gof(x)$ : $4 \xrightarrow{f(x)} 5 \longrightarrow \times$

$6 \longrightarrow 5 \longrightarrow \times$

$8 \longrightarrow 12 \longrightarrow \times$

$10 \longrightarrow 12 \longrightarrow \times$

$\boxed{gof(x) = \varnothing}$

ج) $fof(x)$ : $4 \xrightarrow{f(x)} 5 \longrightarrow \times$

$6 \xrightarrow{f} 5 \longrightarrow \times$

$8 \xrightarrow{f} 12 \longrightarrow \times$

$10 \xrightarrow{f} 12 \longrightarrow \times$

$\boxed{fof(x) = \varnothing}$

د) $gog(x)$ : $4 \xrightarrow{g(x)} 6 \xrightarrow{g(x)} 8$

$2 \longrightarrow 4 \longrightarrow 6$

$6 \longrightarrow 8 \longrightarrow 10$

$8 \longrightarrow 10 \longrightarrow \times$

$gog(x) = \{(4,8),(2,6)(6,10)\}$

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سوال ۵) ۵

$f(g(4)) = 2 \Rightarrow \square \xrightarrow{g(x)} 3 \xrightarrow{f(x)} 2 \rightarrow \square = a = 4$

$g(f(4)) = 1 \Rightarrow 4 \xrightarrow{f(x)} 5 \xrightarrow{g(x)} 1 \rightarrow (3,1)$

$\rightarrow (b,1) \rightarrow \boxed{b=5}$

$(a,b) \Rightarrow (4,5)$