

19,0

«نه هدا، رانی» «باز هم دفتر» «شما» «تکلیف»

$$f(x) = \begin{cases} \cot \frac{\pi x}{\varepsilon}, & x < 1 \rightarrow \cot \frac{\pi \times \frac{1}{\varepsilon}}{\varepsilon} = \cot \frac{\pi}{\varepsilon} = \sqrt{3} \\ \sqrt{x^2 + 1}, & x > 1 \rightarrow f(\sqrt{3}) = \sqrt{(\sqrt{3})^2 + 1} = 2 \end{cases}$$

$f(f(\frac{1}{\varepsilon})) = 2$

$$f\left(\frac{1}{x}\right) = \sqrt{\frac{x-1}{x^2}} \rightarrow f\left(\frac{1}{\frac{1}{\varepsilon}}\right) = f(\varepsilon) = \sqrt{\frac{\varepsilon-1}{\varepsilon^2}} = \frac{\sqrt{\varepsilon}}{\varepsilon}$$

$$g(x) = 2 \cos^2 x \rightarrow g\left(\frac{\pi}{\varepsilon}\right) = 2 \cos^2 \frac{\pi}{\varepsilon} = 2 \times \left(\frac{1}{\varepsilon}\right)^2 = \frac{1}{\varepsilon}$$

$f\left(g\left(\frac{\pi}{\varepsilon}\right)\right) = \frac{\sqrt{\varepsilon}}{\varepsilon}$

$$f(x) = [x] \quad g(x) = \frac{x}{1-x} \quad f(g(\sqrt{2})) = -4$$

$$g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = -\sqrt{2}-2 \rightarrow f(-\sqrt{2}-2) = [-\sqrt{2}-2] = -4$$

$$f(x) = \sin x \quad g(x) = x \sqrt{1-x^2} \quad g\left(f\left(\frac{\pi}{\varepsilon}\right)\right) =$$

$$f\left(\frac{\pi}{\varepsilon}\right) = \sin \frac{\pi}{\varepsilon} = \frac{\sqrt{2}}{\varepsilon} \quad g\left(\frac{\sqrt{2}}{\varepsilon}\right) = \frac{\sqrt{2}}{\varepsilon} \sqrt{1 - \frac{2}{\varepsilon^2}} = \frac{\sqrt{2}}{\varepsilon} \times \frac{1}{\sqrt{\varepsilon}} = \frac{1}{\varepsilon}$$

$$f(x) = \{(\varepsilon, \omega), (4, \omega), (1, 12), (10, 12) \}$$

$$g(x) = \{(\varepsilon, 4), (2, 4), (4, 1), (1, 1) \}$$

$$f \circ g(x) = \{(\varepsilon, \omega), (2, \omega), (4, 12), (1, 12) \}$$

$$g \circ f(x) = \emptyset$$

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$$g \circ g(x) = \{(\varepsilon, 1), (2, 4), (4, 1) \}$$

$$(\varepsilon, 2) \in f \circ g \rightarrow f(g(\varepsilon)) = 2 \rightarrow g(\varepsilon) = 4 \rightarrow a = \varepsilon$$

$$(\varepsilon, 1) \in g \circ f \rightarrow g(f(\varepsilon)) = 1 \rightarrow f(\varepsilon) = \omega, g(\omega) = 1 \rightarrow b = \omega$$

$$(a, b) = (\varepsilon, \omega)$$

$f(x) = \begin{cases} x^2 + 1 & x \geq 1 \\ 2x - 1 & x < 1 \end{cases}$, $g(x) = \begin{cases} x^2 - 1 & x \geq 1 \\ 2x - 1 & x < 1 \end{cases}$, $g \circ f(-1) = ?$ -4
 $a(ax+b)+b \rightarrow a^2x + (ab+b) = x^2 + 1 \rightarrow a=1, b=1 \rightarrow 2x+1$
 $a=-1, b=-1 \rightarrow 2x-1$
 $g \circ f(-1) \rightarrow \begin{cases} \frac{1}{1} (2x+1) - \frac{1}{1} \rightarrow -1 \\ \frac{1}{1} (-2x-1) - \frac{1}{1} \rightarrow -1 \end{cases}$ $g \circ f(-1) = -1$ (5)

$f(x) = \sqrt{x+1}$, $g(x) = \frac{1}{x^2 - 5x}$, $D_{g \circ f} = ?$ -4

$\frac{1}{(\sqrt{x+1})(\sqrt{x+1}-5)} \rightarrow \frac{1}{-1+} \rightarrow D_{g \circ f} = (1, +\infty) \cup \{1\}$ (5)

$f(x) = \sqrt{1-x^2}$, $g(x) = \sqrt{x}$, $(f+g) \circ f = ?$ (1,0) -1

$\sqrt{x} + \sqrt{1-x^2} \rightarrow \sqrt{x^2} + \sqrt{1-x^2} \rightarrow \frac{-1}{-1+1} \rightarrow [-1, 1]$
 $Dg = x \geq 0$, $Df = -1 \leq x \leq 1 \rightarrow [0, 1]$
 $\leq \sqrt{1-x^2} \leq 1 \rightarrow x \geq 0$

الف) $f\left(\frac{x+1}{x-1}\right) = 2x+1 \rightarrow \frac{x+1}{x-1} = t \rightarrow (f+g) \circ f = [1, 1]$ -9

$t = \frac{x+1}{x-1} \rightarrow t(x-1) = x+1 \rightarrow tx - t = x+1 \rightarrow x(t-1) = t+1 \rightarrow x = \frac{t+1}{t-1}$

$f\left(\frac{x+1}{x-1}\right) = 2\left(\frac{t+1}{t-1}\right) + 1 \rightarrow f(t) = \frac{1}{t-1} \rightarrow f(x) = \frac{1}{x-1}$

ب) $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} \rightarrow f(t) = t^2 - 1$ (5)

$x^2 + \frac{1}{x^2} = t^2 - 1 \rightarrow f(x) = x^2 - 1$

$f(x) = x\sqrt{x}$ / $g \circ f$: $(\sqrt{2}, 0), (1, 0)$ -10

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$y = 1 \rightarrow 1$

$x\sqrt{x} = 1 \rightarrow x^2 = 1$

$\begin{cases} f(x) = 1 \\ f(x) = \sqrt{2} \\ x\sqrt{x} = \sqrt{2} \\ x = 2 \end{cases}$ (5)