نام و نام خانوادگی ......حلما عطاری...... پاسخنامه تشریحی تکلیف شماره ...۹... کلاس ......یازدهم ریاضی B......

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$f(x)=\begin{cases}\cot\frac{\pi x}{4} ; & x\le 1\\ \sqrt{x^2+1} ; & x>1\end{cases}$

$fof(\frac{2}{3})=?$

$f(\frac{2}{3})\xrightarrow{\frac{2}{3}<1}\cot\frac{\pi\times 2}{4\times 3}=\cot\frac{\pi}{6}=\sqrt{3}$

$fof(\frac{2}{3})$

$f(f(\frac{2}{3}))=f(\sqrt{3})\xrightarrow{\sqrt{3}>1}\sqrt{\underbrace{(\sqrt{3})^2}_{3}+1}=\sqrt{4}=2$

$fof(\frac{2}{3})=2$

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الف) اگر $f(\frac{1}{x})=\sqrt{\frac{2x-1}{x^2}}$ و $g(x)=2\cos^2 x$ مقدار $fog(\frac{\pi}{3})$ ؟

$f(\frac{1}{x})=\sqrt{\frac{2}{x}-\frac{1}{x^2}}\to f(t)=\sqrt{2t-t^2}$

$fog(x)=f(g(x))=\sqrt{\underbrace{2(2\cos^2x)}_{4\cos^2x}-\underbrace{(2\cos^2x)^2}_{4\cos^4x}}=\sqrt{4\cos^2x(\underbrace{1-\cos^2x}_{\sin^2x})}$

$fog(\frac{\pi}{3})=\sqrt{\cancel{4}\times\frac{1}{\cancel{4}}\times\frac{3}{4}}=\frac{\sqrt{3}}{2}$

ب) $fog\sqrt{2}=?$ $g(x)=\frac{x}{1-x}$ ، $f(x)=[x]$

$fogx=[\frac{x}{1-x}]=[\frac{x}{1-x}+1-1]=[\frac{\cancel{x}+1-\cancel{x}}{1-x}]-1=[\frac{1}{1-x}]-1\xrightarrow{fog\sqrt{2}}[\frac{1}{1-\sqrt{2}}]-1=[\frac{1+\sqrt{2}}{-1}]-1=[-2.4]-1=-4$

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$g(x)=x\sqrt{1-x^2}$, $f(x)=[x]$

$gof(x)=\sin x\sqrt{\underbrace{1-\sin^2x}_{\cos^2x}}=\sin x|\cos x|\to gof(\frac{\pi}{4})=\sin\frac{\pi}{4}|\cos\frac{\pi}{4}|=\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2}=\frac{1}{2}$

$f(\frac{\pi}{4})=[\pi$

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الف) $fog(x):\{(4,5),(2,5),(6,12),(8,12)\}$

ب) $gof(x)=\emptyset$ تهی

ج) $fof(x):\emptyset$ تهی

د) $gog(x):\{(4,8),(2,6),(6,10)\}$

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$f=\{(2,1),(3,2),(4,5),(1,7)\}$ $g=\{(1,2),(3,1),(a,3),(b,1)\}$ $(4,2)\in fog$ - $(4,1)\in gof$ $(a,b)=?$

$fog=\{(1,1)(3,7)(a,2),(b,7)\}\xrightarrow{(4,2)\in fog}\boxed{a=4}$

$(a,b)=(4,5)$

$gof=\{(3,2),\underbrace{(4,1)}_{\text{طبق فرض سؤال}}\}\to gof(4)=4\to\underbrace{5}_{b}\to 1\to\boxed{b=5}$

$f(f(x)) = 4x+3$ $g(\overbrace{2x+3}^{t}) = 3x-2$ $gof(-1) = ?$ f و g توابع خطی $t=2x+3 \to x=\frac{t-3}{2} \to g(t) = 3\left(\frac{t-3}{2}\right)-2 = \frac{3t-9-4}{2} = \frac{3t-13}{2}$ $f(x)=ax+b \to f(f(x)): a(ax+b)+b \to f(f(x)): a^2x+ab+b = 4x+3 \to \begin{cases} a^2=4 \\ a=\pm 2 \end{cases}$ $\begin{cases} a=2 \to 3b=3 \to b=1 \to f(x)=2x+1 \to f(-1) = -2+1 = -1 \\ a=-2 \to -b=3 \to b=-3 \to f(x) = -2x-3 \to f(-1) = -2(-1)-3 = -1 \end{cases}$, $g(x) = \frac{3x-13}{2}$ $gof(-1) = g(f(-1)) = g(-1) = \frac{-3-13}{2} = \frac{-16}{2} = -8$	6
$g(x) = \frac{1}{x^2-4x}$, $f(x) = \sqrt{x+\|x\|} \geq 0$ $D_{gof} = ?$ $D_{gof} = \{\overset{\mathrm{II}:}{x\in D_f} : \overset{\mathrm{I}:}{f(x)\in D_g}\}$ $D_{gof}: \mathrm{I}\cap\mathrm{II}: \mathbb{R}^+ - \{8\}$ $x\in D_f$: $f(x) = \sqrt{x+\|x\|}\geq 0 \to D_f: \mathbb{R} \to x\in D_f$ ✓ برقرار $f(x)\in D_g$: $g(x): \frac{1}{x^2-4x} \neq 0 \to D_g: \mathbb{R}-\{0,4\} \to \begin{cases} \sqrt{x+\|x\|}\neq 0 \to \begin{cases} x\neq 0 \\ x\notin\mathbb{R}^- \end{cases} \to x\in\mathbb{R}^+ \\ \sqrt{x+\|x\|}\neq 4 \xrightarrow{(\,)^2} x+\|x\|\neq 16 \to \begin{cases} +: x+x\neq 16 \to x\neq 8 \\ -: x-x\neq 16 \to 0\neq 16 ✓ \end{cases} \end{cases}$	7
$f(x) = \sqrt{1-x^2}\geq 0 \to x=\pm 1$ $\begin{array}{c\|ccccc} x & & -1 & & +1 & \\ \hline & - & 0 & + & 0 & - \end{array}$ $D_f: [-1,1]$ $g(x) = \sqrt{x}\geq 0 \to D_g = [0,1]$ $D_{f+g}: D_f\cap D_g = [0,1]$ $D_{(f+g)of}: \{x\in D_f : f(x)\in D_{f+g}\} \to \{\overset{:\mathrm{I}}{x\in[-1,1]} , \overset{:\mathrm{II}}{\sqrt{1-x^2}\in[0,1]}\}$ $0\leq\sqrt{1-x^2}\leq 1 \xrightarrow{(\,)^2} 0\leq 1-x^2\leq 1 \xrightarrow{-1} -1\leq -x^2\leq 0 \xrightarrow{\times(-)} 1\geq x^2\geq 0 \to \begin{cases} 1\geq x\geq 0 : ☆ \\ -1\leq x\leq 0 : ☆☆ \end{cases}$ $\mathrm{II}: ☆ \cup ☆☆ = -1\leq x\leq 1$ $D_{(f+g)of}: \mathrm{I}\cap\mathrm{II}: [-1,1]$	8
الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5 \to f(t) = \frac{4(2t+1)}{t-3}+5 = \frac{8t+4+5t-15}{t-3} = \frac{13t-11}{t-3} \to f(x) = \frac{13x-11}{x-3}$ $\frac{3x+1}{x-2} = t \to x = -\frac{-2t-1}{t-3} = \frac{2t+1}{t-3}$ ب) $f(\overbrace{x+\frac{1}{x}}^{t}) = x^3+\frac{1}{x^3} \to f(x) = x^3-3x$ $x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)\left(x^2-1+\frac{1}{x^2}\right) = (t)(\overbrace{t^2-2}^{}-1) \to f(t) = (t)(t^2-3) \to f(t) = t^3-3t$ $t^2 = \left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2 \to x^2+\frac{1}{x^2} = t^2-2$	9
$g(x) = (x-1)(x-2\sqrt{2})$ نمودار g محور xها را در نقاطی به طول ۱ و ۲√۲ قطع می‌کند: $f(x) = x\sqrt{x}$ $gof(x) = (x\sqrt{x}-1)(x\sqrt{x}-2\sqrt{2}) \to \begin{cases} x\sqrt{x}-1=0 \to x\sqrt{x}=1 \to x=1 \\ x\sqrt{x} = 2\sqrt{2} \to x=2 \end{cases}$ طول نقاطی که محور xها را قطع می‌کند: اختلاف طول نقاط: $2-1 = 1$	10