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$$f \circ f\left(\frac{r}{r}\right) = ?$$

$$f(x) = \begin{cases} \cot \frac{\pi x}{r} ; x \leq 1 \\ \sqrt{x^2 + 1} ; x > 1 \end{cases}$$

$$\frac{r}{r} \leq 1 \rightarrow \cot \frac{\pi \times \frac{r}{r}}{r} = \cot \frac{\pi}{r} = \sqrt{r}$$

$$f \circ f\left(\frac{r}{r}\right) = f(\sqrt{r}) \xrightarrow{\sqrt{r} > 1} \sqrt{(\sqrt{r})^2 + 1} = \sqrt{r + 1} = r$$

$$f \circ f\left(\frac{r}{r}\right) = r$$

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$$f\left(\frac{r}{r}\right) = \sqrt{\frac{r}{r} - \frac{1}{r}} \rightarrow f(t) = \sqrt{rt - t^2} \quad \log(x) = f(g(x)) = \sqrt{x(r \cos^2 x) - (\cos^2 x)^2} = \sqrt{r \cos^2 x (1 - \cos^2 x)} = \sqrt{r \cos^2 x \sin^2 x} = \sqrt{r} \cos x \sin x$$

$$f \circ g\left(\frac{r}{r}\right) = \sqrt{r \times \frac{1}{r} \times \frac{r}{r}} = \sqrt{\frac{r}{r}} = \sqrt{r}$$

$$f \circ g \sqrt{r} = ?$$

$$g(x) = \frac{x}{1-x} \quad f(n) = [n]$$

$$f \circ g x = \left[\frac{x}{1-x} \right] = \left[\frac{x}{1-x} + 1 - 1 \right] = \left[\frac{x+1-x}{1-x} \right] - 1 = \left[\frac{1}{1-x} \right] - 1 \xrightarrow{f \circ g \sqrt{r}} \left[\frac{1}{1-\sqrt{r}} \right] - 1 = \left[\frac{1+\sqrt{r}}{1-\sqrt{r}} \right] - 1 = \left[\frac{1+\sqrt{r}}{1-\sqrt{r}} - \frac{1-\sqrt{r}}{1-\sqrt{r}} \right] = \left[\frac{2\sqrt{r}}{1-\sqrt{r}} \right] = -r$$

$$g(x) = x\sqrt{1-x^2}, \quad f(n) = [n]$$

$$g \circ f(x) = \sin x \sqrt{1 - \sin^2 x} = \sin x |\cos x| \rightarrow g \circ f\left(\frac{\pi}{r}\right) = \sin \frac{\pi}{r} \left| \cos \frac{\pi}{r} \right| = \frac{\sqrt{r}}{r} \times \frac{\sqrt{r}}{r} = \frac{1}{r}$$

$$f\left(\frac{\pi}{r}\right) = [r]$$

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$$f \circ g(x) : \{(r, 0), (r, 1), (4, 1), (8, 1)\}$$

$$g \circ f(x) = \emptyset \text{ or}$$

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$$f \circ g(x) : \emptyset \text{ or}$$

$$g \circ g(x) : \{(r, 8), (r, 4), (4, 1)\}$$

$$f = \{(r, 1), (r, 2), (r, 3), (b, 1)\} \quad g = \{(r, 1), (r, 2), (a, 1), (b, 1)\} \quad (r, 1) \in f \circ g - (r, 1) \in g \circ f \quad (a, b) = ?$$

$$f \circ g = \{(1, 1), (r, 1), (a, 1), (b, 1)\} \quad (r, 1) \in f \circ g \quad a = r$$

$$(a, b) = (r, 2)$$

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$$g \circ f = \{(r, 1), (r, 2)\} \rightarrow g \circ f(r) = r \rightarrow \frac{a}{b} \rightarrow 1 \rightarrow \boxed{b = a}$$

طبق فرض سوال

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$f(f(n)) = rx + r$ $g(\frac{t}{r}) = rn - r$ $g \circ f(-1) = ?$ $g \circ f$ تابع خطی
 $t = rn + r \rightarrow n = \frac{t-r}{r} \rightarrow g(t) = r(\frac{t-r}{r}) - r = \frac{rt - r - r}{r} = \frac{rt - 2r}{r}$
 $f(x) = ax + b \rightarrow f(f(n)) : a(ax + b) + b \rightarrow f(f(n)) : a^2x + a(b + b) = rx + r \rightarrow \begin{cases} a^2 = r \\ a(b + b) = r \end{cases}$
 $\begin{cases} a = r \\ a = -r \end{cases} \rightarrow \begin{cases} rb = r \rightarrow b = 1 \\ -b = r \rightarrow b = -r \end{cases} \rightarrow f(n) = rn + 1 \rightarrow f(-1) = -r + 1 = -1$
 $a = -r \rightarrow -b = r \rightarrow b = -r \rightarrow f(n) = -rn - r \rightarrow f(-1) = -r(-1) - r = -1$ $g(n) = \frac{rn - 1}{r}$
 $g \circ f(-1) = g(f(-1)) = g(-1) = \frac{-r - 1}{r} = \frac{-14}{r} = -1$

$g(x) = \frac{1}{x^2 - 5x}$, $f(x) = \sqrt{x+|x|}$, $D_{g \circ f} = ?$
 $D_{g \circ f} = \{x \in D_f : f(x) \in D_g\}$ $D_{g \circ f} = I \cap II : \mathbb{R}^+ - \{1\}$
 $\begin{cases} x \in D_f & f(x) = \sqrt{x+|x|} \geq 0 \rightarrow D_f : \mathbb{R} \rightarrow x \in D_f \checkmark \\ f(x) \in D_g & g(x) = \frac{1}{x^2 - 5x} \neq 0 \rightarrow D_g : \mathbb{R} - \{0, 5\} \end{cases}$
 $\rightarrow \begin{cases} \sqrt{x+|x|} \neq 0 \rightarrow \begin{cases} x \neq 0 \\ x \in \mathbb{R}^+ \end{cases} \rightarrow x \in \mathbb{R}^+ \\ \sqrt{x+|x|} \neq 5 \rightarrow x+|x| \neq 25 \rightarrow \begin{cases} x+x \neq 25 \\ x-x \neq 25 \end{cases} \rightarrow \begin{cases} x \neq 12.5 \\ x \neq 12.5 \end{cases} \end{cases}$

$f(x) = \sqrt{1-x^2} \geq 0 \rightarrow x = \pm 1$ $\frac{x}{1-x^2} = \frac{x}{-x^2+1} \rightarrow D_f : [-1, 1]$
 $g(x) = \sqrt{x} \geq 0 \rightarrow D_g = [0, 1]$ $D_{f+g} : D_f \cap D_g = [0, 1]$
 $D_{(f+g) \circ f} = \{x \in D_f : f(x) \in D_{f+g}\} \rightarrow \begin{cases} I : x \in [-1, 1] \\ II : \sqrt{1-x^2} \in [0, 1] \end{cases}$
 $\begin{cases} \sqrt{1-x^2} \leq 1 \rightarrow 1-x^2 \leq 1 \rightarrow -1 \leq -x^2 \leq 0 \rightarrow 1 \geq x^2 \geq 0 \rightarrow \begin{cases} 1 \geq x^2 \geq 0 : \star \\ -1 \leq x \leq 0 : \star \star \end{cases} \end{cases}$
 $D_{(f+g) \circ f} : I \cap II : [-1, 1]$ $II : \star \cup \star \star = -1 \leq x \leq 1$

$الف) f(\frac{rt+1}{t-r}) = rn + a \rightarrow f(t) = \frac{f(rt+1)}{t-r} + a = \frac{rt+1}{t-r} + a = \frac{rt+1+a(t-r)}{t-r} = \frac{rt+1+at-rt}{t-r} = \frac{at-1}{t-r}$
 $\frac{rt+1}{t-r} = t \rightarrow x = -\frac{rt-1}{t-r} = \frac{rt+1}{t-r}$
 $\rightarrow f(\frac{t}{x+\frac{1}{x}}) = x^r + \frac{1}{x^r} \rightarrow f(n) = x^r - rx$
 $x^r + \frac{1}{x^r} = (x + \frac{1}{x})(x^r - 1 + \frac{1}{x^r}) = (t)(t^r - 1) \rightarrow f(t) = (t)(t^r - 1) \rightarrow f(t) = t^r - rt$
 $t^r = (x + \frac{1}{x})^r = x^r + \frac{1}{x^r} + r \rightarrow x^r + \frac{1}{x^r} = t^r - r$

$g(n) = (n-1)(n-r\sqrt{r})$ g تابع خطی و g تابع زوج
 $f(n) = n\sqrt{n}$
 $g \circ f(n) = (n\sqrt{n} - 1)(n\sqrt{n} - r\sqrt{r}) \rightarrow \begin{cases} n\sqrt{n} - 1 = 0 \rightarrow n\sqrt{n} = 1 \rightarrow n = 1 \\ n\sqrt{n} - r\sqrt{r} = 0 \rightarrow n = r \end{cases}$
 $دو جواب : r-1 = 1$