۲۰

نام و نام خانوادگی ......علینا کریمی...... پاسخنامه تشریحی تکلیف شماره ...۹... کلاس ...یازدهم تجربی C...

۱

$$f(x)=\begin{cases}\cot\frac{\pi x}{4} & ; x\leq 1\\ \sqrt{x^2+1} & ; x>1\end{cases}$$

$f(\frac{2}{3})=\cot\frac{\pi\frac{2}{3}}{4}=\cot\frac{\pi}{6}=\sqrt{3}$

$\Rightarrow fof(\frac{2}{3})=f(\sqrt{3})=\sqrt{(\sqrt{3})^2+1}=\boxed{2}$

۲

۲

الف) $f(\frac{1}{x})=\sqrt{\frac{2x-1}{x^2}}$ $\quad g(x)=2\cos^2 x$

$g(\frac{\pi}{3})=2\cos^2\frac{\pi}{3}=\frac{1}{2}\rightarrow fog=f(\frac{1}{2})=\sqrt{\frac{2\times 2-1}{2^2}}=\boxed{\frac{\sqrt{3}}{2}}$

۲

ب) $f(x)=[x]$ $\quad g(x)=\frac{x}{1-x}$

$\Rightarrow fog(x)=[\frac{x}{1-x}]=[\frac{x-1+1}{1-x}]=[\frac{1}{1-x}]-1\longrightarrow fog(\sqrt{2})=[\frac{1}{1-\sqrt{2}}]-1=[-\sqrt{2}-1]-1$

$\Rightarrow fog(\sqrt{2})=[-\sqrt{2}]-2=\boxed{-4}$

۳

$f(x)=\sin x$

$g(x)=x\sqrt{1-x^2}$

$f(\frac{\pi}{4})=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

۲

$gof(\frac{\pi}{4})=g(\frac{\sqrt{2}}{2})=\frac{\sqrt{2}}{2}\sqrt{1-(\frac{\sqrt{2}}{2})^2}=\frac{\sqrt{2}}{2}\times\frac{1}{\sqrt{2}}\Rightarrow\boxed{fog(\frac{\pi}{4})=\frac{1}{2}}$

۴

$f(x)=\{(4,5)(6,5)(8,12)(10,12)\}$

$g(x)=\{(4,6)(2,4)(6,8)(8,10)\}$

الف) $fog(x)=\{(4,5)(2,5)(6,12)(8,12)\}$

۲

ب) $gof(x)=\varnothing$

ج) $fof(x)=\varnothing$

د) $gog(x)=\{(4,8)(2,6)(6,10)\}$

۵

$f=\{(2,1),(3,2),(4,5),(1,7)\}$

$g=\{(1,2),(3,1),(a,3),(b,1)\}$

$(4,2)\in fog\longrightarrow f(x)=2\rightarrow x=3\rightarrow f(y)=\underset{\sim}{3}\rightarrow\boxed{y=a=4}$

۲

$(4,1)\in gof\longrightarrow f(4)=5\rightarrow f(5)=1\rightarrow\boxed{5=b}$

$\Rightarrow\boxed{(a,b)=(4,5)}$

$f(f(x)) = 4x+3 \qquad g(2x+3) = 3x-2$

$f(x) = ax+b \rightarrow f(f(x)) = a^2x+ab+b = 3+4x \rightarrow a^2 = 4 \rightarrow a = \pm 2$

$\rightarrow a=2 \rightarrow b=1 \rightarrow f(x) = 2x+1 \Rightarrow f(-1) = -1$

$\searrow a=-2 \rightarrow b=-3 \rightarrow f(x) = -2x-3$ (۳)

$g\circ f(-1) = g(-1) \rightarrow -1 = 2x+3 \rightarrow x=-2 \rightarrow g(-1) = 3\times(-2)-2 = \boxed{-8}$ ۶

$f(x) = \sqrt{x+|x|} \qquad g(x) = \dfrac{1}{x^2-4x}$

$D_{g\circ f} = \{x \in D_f : f(x) \in D_g\}$ (۲)

$D_f: \ x+|x| \geq 0 \rightarrow x \in \mathbb{R}$ همواره

$D_g: \ x \neq 0,4 \rightarrow f(x) \neq 0 \rightarrow x \in \mathbb{R}^+$; $f(x) \neq 4 \rightarrow x \neq 8$ $\xrightarrow{\cap} \boxed{D_{g\circ f} = \mathbb{R}^+ - \{8\}}$ ۷

$f(x) = \sqrt{1-x^2} \qquad g(x) = \sqrt{x}$

$D_f: -1 \leq x \leq 1 \qquad D_{f+g} = D_f \cap D_g = 0 \leq x \leq 1$ (۳)

$D_g: \ x \geq 0$

$D_{(f+g)\circ f}: \ \{x \in D_f,\ f(x) \in D_{f+g}\} \rightarrow -1 \leq x \leq 1$ ① ۸

$0 \leq \sqrt{1-x^2} \leq 1 \rightarrow 1-x^2 \leq 1 \rightarrow x^2 \geq 0$ همواره برقرار ②

$\Rightarrow \boxed{D_{(f+g)\circ f} = [-1,1]}$

الف) $f\left(\dfrac{3x+1}{x-2}\right) = 4x+5$

$\dfrac{3x+1}{x-2} = t \rightarrow x = \dfrac{2t+1}{t-3} \Rightarrow f(t) = 4\left(\dfrac{2t+1}{t-3}\right)+5 = \dfrac{13t-11}{t-3} \Rightarrow \boxed{f(x) = \dfrac{13x-11}{x-3}}$

(۲) ب) $f\left(x+\dfrac{1}{x}\right) = x^3+\dfrac{1}{x^3} = \left(x+\dfrac{1}{x}\right)\left(x^2+\dfrac{1}{x^2}-1\right) = \left(x+\dfrac{1}{x}\right)\left(\left(x+\dfrac{1}{x}\right)^2-3\right)$ ۹

$x+\dfrac{1}{x} = t, \quad f(t) = t(t^2-3) \rightarrow f(t) = t^3-3t \rightarrow \boxed{f(x) = x^3-3x}$

$g(x) = a(x-1)(x-2\sqrt{2})$

$f(x) = x\sqrt{x}$

$g\circ f(x) = a(x\sqrt{x}-1)(x\sqrt{x}-2\sqrt{2}) \xrightarrow[\text{شد}]{\text{ریشه‌های تابع}} \begin{array}{l} x\sqrt{x}-1=0 \rightarrow x=1 \\ x\sqrt{x}-2\sqrt{2}=0 \rightarrow x=2 \end{array}$ (۲) ۱۰

$\Rightarrow 2-1 = \boxed{1}$