باران کاظمی یازدهم C تکلیف شماره ۹

۱-

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & ; x \le 1 \\ \sqrt{x^2+1} & ; x > 1 \end{cases} \qquad f(f(\tfrac{2}{3}))$$

$$f(\tfrac{2}{3}) = \cot \frac{\frac{2}{3}\pi}{4} = \cot \frac{\pi}{6} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$f(\sqrt{3}) = \sqrt{4} = 2$$

۲-

الف)

$$f(\tfrac{1}{x}) = \sqrt{\frac{2x-1}{x^2}} \qquad g(x) = 2\cos^2 x \rightarrow 2\cos^2\frac{\pi}{3} = \frac{1}{2}$$

$$f(g(\tfrac{\pi}{3}))$$

$$\frac{1}{2} = \frac{1}{x} \rightarrow x = 2 \qquad \sqrt{\frac{4-1}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$$

ب)

$$f(x) = [x] \qquad g(x) = \frac{x}{1-x}$$

$$f(g(\sqrt{2})) \qquad \frac{\sqrt{2}}{1-\sqrt{2}} \qquad f(\frac{\sqrt{2}}{1-\sqrt{2}}) = [\frac{\sqrt{2}}{1-\sqrt{2}}] = [-\frac{1.4}{0.4}] = [-3.5] = -4$$

۳-

$$f(x) = \sin x \qquad g(x) = x\sqrt{1-x^2} \qquad g(f(\tfrac{\pi}{4}))$$

$$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$g(\tfrac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\sqrt{1-\frac{2}{4}} = \frac{\sqrt{2} \times \sqrt{2}}{2 \times 2} = \frac{1}{2}$$

۴-

$$f(x) = \{(4,5)(6,5)(8,12)(10,12)\} \qquad g(x) = \{(4,6)(2,4)(6,8)(8,10)\}$$

الف) $fog(x) = \{(4,5)(2,5)(6,12)(8,12)\}$

ب) $gof(x) = \varnothing$

ج) $fof(x) = \varnothing$

د) $gog(x) = \{(4,8)(2,6)(6,10)\}$

۵-

$$f = \{(2,1)(3,2)(4,5)(1,7)\} \qquad a = 4$$

$$g = \{(1,2)(3,1)(a,3)(b,1)\} \qquad b = 5$$

$$(4,2) \in fog \qquad (4,1) \in gof \qquad (4,5)$$

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$f(x)=ax+b$ $\quad a(ax+b)+b=4x+3$

$a^2x+ab+b=4x+3$ $\quad a^2=4 \begin{cases} \to a=2 \to b=1 \\ \to a=-2 \to b=-3 \end{cases}$

$ab+b=3 \xrightarrow{a=2} 3b+b=3 \Rightarrow b=-3$ $\quad y=2x+1$

$b(a+1)=3$ $\quad y=-2x-3$

$2x+1=t \to x=\frac{t-1}{2} \Rightarrow 3\left(\frac{t-1}{2}\right)-2=\frac{3t}{2}-\frac{3}{2}-2=\frac{3x-3-4}{2}=\frac{3x-7}{2}$

$f(-1)=-1$

$g(-1)=\frac{-3-7}{2}=-5$ (گزینه ۱)

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$f(x)=\sqrt{x+|x|}$ $\quad D_f: x+|x|\geq 0$ $\quad x\in D_f$

$g(x)=\frac{1}{x^2-4x}$ $\quad |x|\geq -x$ $\quad f(x)\in D_g$ $\quad D_g=R-\{0,4\}$

$D_{gof}=\mathbb{R}-\{0,4\}$

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$f(x)=\sqrt{1-x^2}$ $\quad g(x)=\sqrt{x}$ $\quad x\geq 0$ $\quad x\in D_f$ $\quad f(x)\in D_g$

$fof+gof$ $\quad \frac{(1-x)(1+x)}{-1\ \ 1}$ $\quad [-1,1]$

$D_{(f+g)of}=[0,1]$

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الف) $f\left(\frac{2x+1}{x-2}\right)=4x+5$ $\quad \frac{2x+1}{x-2}=t \Rightarrow 2x+1=tx-2t$

$x(t-2)=2t+1 \Rightarrow x=\frac{2t+1}{t-2}$

$4\left(\frac{2t+1}{t-2}\right)+5=\frac{8t+4}{t-2}+5=\frac{8t+4+5t-10}{t-2}=\frac{13t-11}{t-2}=\boxed{\frac{13x-11}{x-2}}$

ب) $f\left(x+\frac{1}{x}\right)=x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)^{x^3-x}$

$x\left(\left(x+\frac{1}{x}\right)^2-3\right)=\boxed{x^3-3x}$

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$x\sqrt{x}=2\sqrt{2} \Rightarrow x=2$ $\quad \left.\begin{matrix} x\sqrt{x}=1 \Rightarrow x=1 \end{matrix}\right\} \xrightarrow{\ominus} 2-1=(1)$