باران کاظمی　یازدهم C　تکلیف شماره ۹

۱۸٫۲۵

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$$f(x)=\begin{cases}\cot\frac{\pi x}{4} & ; x\le 1\\ \sqrt{x^2+1} & ; x>1\end{cases}\qquad f(f(\tfrac{2}{3}))$$

$$f(\tfrac{2}{3})=\cot\frac{\frac{2}{3}\pi}{4}=\cot\frac{\pi}{6}=\frac{\frac{\sqrt3}{2}}{\frac12}=\sqrt3$$

$$f(\sqrt3)=\sqrt4=2$$

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۲- الف)

$$f(\tfrac{1}{x})=\sqrt{\frac{2x-1}{x^2}}\qquad g(x)=2\cos^2x\to 2\underbrace{\cos^2\frac{\pi}{3}}_{\frac14}=\frac12$$

$$f(g(\tfrac{\pi}{3}))$$

$$\frac12=\frac1x\to x=2\qquad \sqrt{\frac{4-1}{4}}=\boxed{\sqrt{\frac34}}=\frac{\sqrt3}{2}$$

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ب)

$$f(x)=[x]\qquad g(x)=\frac{x}{1-x}$$

$$f(g(\sqrt2))\qquad \frac{\sqrt2}{1-\sqrt2}\qquad f\left(\frac{\sqrt2}{1-\sqrt2}\right)=\left[\frac{\overset{1.4}{\sqrt2}}{\underset{-0.4}{1-\sqrt2}}\right]=\left[-\frac{14}{4}\right]=[-3.5]=\boxed{-4}$$

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$$f(x)=\sin x\qquad g(x)=x\sqrt{1-x^2}\qquad g(f(\tfrac{\pi}{4}))$$

$$\sin\frac{\pi}{4}=\frac{\sqrt2}{2}$$

$$g(\tfrac{\sqrt2}{2})=\frac{\sqrt2}{2}\sqrt{1-\frac{2}{4}}=\frac{\sqrt2\times\sqrt2}{2\times2}=\frac12$$

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۴-

$$f(x)=\{(4,5)(6,5)(8,12)(10,12)\}\qquad g(x)=\{(4,6)(2,4)(6,8)(8,10)\}$$

الف) $fog(x)=\{(4,5)(2,5)(6,12)(8,12)\}$

ب) $gof(x)=\varnothing$

ج) $fof(x)=\varnothing$

د) $gog(x)=\{(4,8)(2,6)(6,10)\}$

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$$f=\{(2,1)(3,2)(4,5)(1,7)\}\qquad a=4$$

$$g=\{(1,2)(3,1)(a,3)(b,1)\}\qquad b=5\qquad (4,5)$$

$$(4,2)\in fog\qquad (4,1)\in gof$$

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$f(x) = ax+b \qquad a(ax+b)+b = 4x+3$ — ۶

$a^2x + ab + b = 4x+3 \qquad a^2 = 4 \begin{cases} a=2 \to b=1 \\ a=-2 \to b=-3 \end{cases}$

$ab+b = 3 \xrightarrow{a=-2} -2b+b=3 \Rightarrow b=-3 \qquad y = 2x+1$

$b(a+1) = 3 \qquad y = -2x-3$

$2x+3 = t \Rightarrow x = \frac{t-3}{2} \Rightarrow 3\left(\frac{t-3}{2}\right) - 2 = \frac{3t}{2} - \frac{9}{2} - 2 = \frac{3x-9-4}{2} = \frac{3x-13}{2}$

$f(-1) = -1$

$g(-1) = \frac{-3-13}{2} = -8$ (۴)

$f(x) = \sqrt{x+|x|} \quad D_f: x+|x| \geq 0 \qquad x \in D_f$

$g(x) = \frac{1}{x^2-4x} \qquad |x| \geq -x \qquad f(x) \in D_g \qquad D_g = R - \{0, 4\}$ — ۷

$\cancel{D_{gof} = R - \{0,4\}} \qquad \sqrt{x+|x|} \neq 0 \to x > 0$

$\sqrt{x+|x|} \neq 4 \to x \neq 8 \qquad \to D_{gof} = (0, +\infty) - \{8\}$ (۱)

$f(x) = \sqrt{1-x^2} \qquad g(x) = \sqrt{x} \qquad x \in D_f$

$fof + gof \qquad (1-x)(1+x) \qquad x \geq 0 \qquad f(x) \in D_g$ — ۸

$-1 \quad 1 \qquad [-1, 1]$

$D_{(f+g)of} = [0, 1]$ (۱,۲۵)

$0 \leq \sqrt{1-x^2} \leq 1 \to x^2 \geq 0 \checkmark$

$D_{(f+g)of} = [-1, 1]$

الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5 \qquad \frac{3x+1}{x-2} = t \Rightarrow 3x+1 = tx-2t$

$x(t-3) = 2t+1 \Rightarrow x = \frac{2t+1}{t-3}$ — ۹

$4\left(\frac{2t+1}{t-3}\right) + 5 = \frac{8t+4}{t-3} + 5 = \frac{8t+4+5t-15}{t-3} = \frac{13t-11}{t-3} = \boxed{\frac{13x-11}{x-3}}$

ب) $f\left(x+\frac{1}{x}\right) = x^3 + \frac{1}{x^3} = \left(\underbrace{x+\frac{1}{x}}_{x}\right)\left(x^2+\frac{1}{x^2}-1\right)^{+2-2}$ (۲)

$x\left(\left(\underbrace{x+\frac{1}{x}}_{x}\right)^2 - 3\right) = \boxed{x^3 - 3x}$

$x\sqrt{x} = 2\sqrt{2} \Rightarrow x=2$
$x\sqrt{x} = 1 \Rightarrow x = 1$ $\xrightarrow{\ominus} 2-1 = 1$ — ۱۰ (۲)