

$$① f\left(\frac{2}{3}\right) = \cot \frac{\pi}{4} = \sqrt{3} \text{ و } f(\sqrt{3}) = \sqrt{4} = 2 \rightarrow f \circ f = 2$$

$$② \text{ الف) } g\left(\frac{\pi}{3}\right) = \frac{1}{\pi} \text{ و } f\left(\frac{1}{\pi}\right) = \sqrt{\frac{\pi}{4}} = \frac{\sqrt{\pi}}{2} \rightarrow f \circ g\left(\frac{\pi}{3}\right) = \frac{\sqrt{\pi}}{2}$$

$$\text{ب) } g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} = \frac{\sqrt{2}(1+\sqrt{2})}{-1} = -\sqrt{2}-2 \rightarrow f(-\sqrt{2}-2) = [-\sqrt{2}-2] = -4$$

$$③ f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{\pi} \text{ و } g\left(\frac{\sqrt{2}}{\pi}\right) = \frac{\sqrt{2}}{\pi} \times \sqrt{\frac{1}{2}} = \frac{1}{\pi} \rightarrow g \circ f\left(\frac{\pi}{4}\right) = \frac{1}{\pi}$$

$$④ \text{ الف) } f \circ g(x) = \{(4, 5), (2, 5), (9, 12), (1, 12)\}$$

$$\text{ب) } g \circ f(x) = \emptyset$$

$$\text{ج) } f \circ f(x) = \emptyset$$

$$\text{د) } g \circ g(x) = \{(4, 1), (2, 9), (9, 10)\}$$

$$⑤ (4, 2) \in f \circ g \rightarrow a = 4 \text{ و } (4, 1) \in g \circ f \rightarrow b = 5 \rightarrow (4, 5)$$

$$⑥ f(x) = ax + b \rightarrow f(ax + b) = a^2x + ab + b = 5x + 3 \rightarrow a = \pm 2 \rightarrow \begin{cases} f(x) = 2x + 1 \\ f(x) = -2x - 3 \end{cases}$$

اگر $x = -1 \rightarrow f(x) = -1 \rightarrow g(-1) = -1 \rightarrow g \circ f(-1) = -1$

$2x + 3 = -1 \rightarrow x = -2$

$$⑦ x \in D_{f(x)} \text{ و } R_{f(x)} \in D_{g(x)} \rightarrow D_{f(x)} = \mathbb{R} \rightarrow \sqrt{x+1}$$

$$D_{g(x)} = \mathbb{R} - \{0, 4\} \rightarrow [0, +\infty) \cap \mathbb{R} - \{0, 4\} = (0, +\infty) - \{4\}$$

ریشه‌های منجم

$R_{f(x)} = [0, +\infty)$

$\sqrt{x+1} \in (0, +\infty) - \{4\}$

$x \in (0, +\infty) - \{1\}$

① $f(x) + g(x) = \sqrt{x} + \sqrt{1-x^2}$, $(f+g) \circ f \rightarrow \sqrt{1-x^2} + \sqrt{x^2}$ وحيث $x \geq 0$
 $1-x^2 \geq 0 \rightarrow \frac{-1}{-2} + \frac{1}{2} \rightarrow \boxed{D_f = [-1, 1]}$ $\leftarrow (f+g) \circ f = \sqrt{1-x^2} + x$

② الف) $\frac{t^2+1}{t-2} = t \rightarrow x = t + \frac{t^2+1}{t-2} \rightarrow f(t) = t + \frac{t^2+1}{t-2} + a \rightarrow f(t) = \frac{t^2+t+1}{t-2} + a$
 $\boxed{f(x) = \frac{13x-11}{x-2}}$ $\leftarrow$

ب) $f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right) \left(\frac{x^2 + \frac{1}{x^2} - 1}{\left(x + \frac{1}{x}\right)^2 - 2} \right) \rightarrow f(x) = x(x^2 - 2) \rightarrow \boxed{f(x) = x^3 - 2x}$

③ $f(x) = 1 \rightarrow x\sqrt{x} = 1 \rightarrow \boxed{x=1}$ $\rightarrow \boxed{2-1=1}$
 $f(x) = 2\sqrt{2} \rightarrow x\sqrt{x} = 2\sqrt{2} \rightarrow \boxed{x=2}$