زهرا احمدی یازدهم دختر تکلیف شماره‌ی ۹ (۱۹٫۵)

① $f(\frac{2}{3}) = \cot\frac{\pi}{4} = \sqrt{3}$ ، $f(\sqrt{3}) = \sqrt{4} = 2 \rightarrow \boxed{fof_{(4)} = 2}$ (۲)

② الف) $g(\frac{\pi}{3}) = \frac{1}{2}$ و $f(\frac{1}{2}) = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} \rightarrow \boxed{fog(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}}$ (۲)

ب) $g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} = \frac{\sqrt{2}(1+\sqrt{2})}{-1} = -\sqrt{2}-2 \rightarrow \boxed{f(-\sqrt{2}-2) = [-\sqrt{2}-2] = -4}$

③ $f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$ ، $g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2} \times \sqrt{\frac{1}{2}} = \frac{1}{2} \rightarrow \boxed{gof(\frac{\pi}{4}) = \frac{1}{2}}$ (۲)

④ الف) $fog(x) = \{(4,5), (2,3), (9,12), (8,12)\}$

ب) $gof(x) = \emptyset$

ج) $fof(x) = \emptyset$

د) $gog(x) = \{(4,8), (2,9), (9,10)\}$ (۲)

⑤ $(4,2) \in fog \rightarrow \boxed{a=4}$ ، $(4,1) \in gof \rightarrow \boxed{b=3} \rightarrow \boxed{(4,3)}$ (۲)

⑥ $f(x) = ax+b \rightarrow f(ax+b) = a^2x + ab + b = 4x + 3 \rightarrow a = \mp 2 \rightarrow$ $\nearrow f(x) = 2x+1$ $\searrow f(x) = -2x-3$

اگر $x=-1 \rightarrow f(x) = -1 \rightarrow g(-1) = -8 \rightarrow \boxed{gof(-1) = -8}$

$2x+3 = -1 \rightarrow \boxed{x=-2}$ ← (۲)

⑦ $x \in D_{f(x)}$ ، $R_{f(x)} \in D_{g(x)} \rightarrow D_{f(x)} = \mathbb{R} \rightarrow \sqrt{x+|x|}$

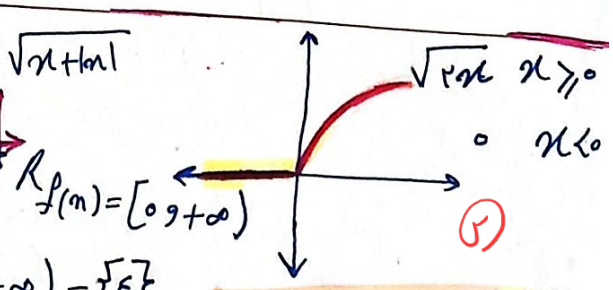
$D_{g(x)} = \mathbb{R} - \{0, 4\} \rightarrow [0,+\infty) \cap \mathbb{R} - \{0,4\} = (0,+\infty) - \{4\}$

ریشه‌های مخرج

$R_{f(x)} = [0,+\infty)$

$\sqrt{x+|x|} \in (0,+\infty) - \{4\}$

$x \in (0,+\infty) - \{8\}$



(۲)

$D_f = -1 \leqslant x \leqslant 1 \cap D_g = x \geqslant 0 \rightarrow [0,1]$ $\quad 0 \leqslant \sqrt{1-x^2} \leqslant 1 \rightarrow x^2 \geqslant 0 \checkmark$

۸) $f(x)+g(x) = \sqrt{x} + \sqrt{1-x^2}$ و $(f+g)of \rightarrow \sqrt{\sqrt{1-x^2}} + \sqrt{x^2}$ $\quad D_{(f+g)of} = [-1,1]$ (۱٫۵)

چون $x \geqslant 0$

$1-x^2 \geqslant 0 \rightarrow$ -1 ــ + ــ $+1$ $\rightarrow D_f = [-1,1] \leftarrow (f+g)of = \sqrt[4]{1-x^2} + x$

---

۹) الف) $\frac{3x+1}{x-2} = t \rightarrow x = +\frac{2t+1}{t-3} \rightarrow f(t) = 4\left(\frac{2t+1}{t-3}\right) + 5 \rightarrow f(t) = \frac{8t+4}{t-3} + 5$

$f(x) = \frac{13x-11}{x-3}$ (۲)

ب) $f\left(x+\frac{1}{x}\right) = \left(x+\frac{1}{x}\right)\underbrace{\left(x^2+\frac{1}{x^2}-1\right)}_{\left(x+\frac{1}{x}\right)^2-3} \rightarrow f(x) = x\left(x^2-3\right) \rightarrow f(x) = x^3 - 3x$

---

۱۰) $f(x) = 1 \rightarrow x\sqrt{x} = 1 \rightarrow x = 1$ $\quad \longrightarrow 2-1 = 1$ (۲)

$f(x) = 2\sqrt{2} \rightarrow x\sqrt{x} = 2\sqrt{2} \rightarrow x = 2$