

کارهای باقی - کمالی ۹۰

نام

دستان یا زهم (C) ۲۰

(1)

$$f(n) = \begin{cases} \cot \frac{\pi n}{r}, & n \leq 1 \\ \sqrt{n^2 + 1}, & n > 1 \end{cases}$$

$$f\left(\frac{r}{r}\right) = \cot \frac{\pi r}{r} = \cot \pi = \sqrt{r}$$

$$f(\sqrt{r}) \rightarrow \sqrt{(\sqrt{r})^2 + 1} = \boxed{2}$$

(۲)

الف) $f \circ g\left(\frac{r}{r}\right) \rightarrow g\left(\frac{r}{r}\right) = r \cos^2\left(\frac{\pi}{r}\right) = \frac{1}{r}$

(2)

(۲)

$$\sqrt{\frac{\frac{r}{n} - 1}{\frac{1}{n^2}}} = \sqrt{\frac{\frac{r-n}{n}}{\frac{1}{n^2}}} = \sqrt{(r-n)(n)} = \sqrt{rn - n^2} \rightarrow f \circ g\left(\frac{r}{r}\right) = \sqrt{1 - \frac{1}{r}}$$

ب) $f \circ g(\sqrt{r}) \rightarrow g(\sqrt{r}) = \frac{\sqrt{r}}{1 - \sqrt{r}} \rightarrow \frac{\sqrt{r}}{1 - \sqrt{r}} \times \frac{1 + \sqrt{r}}{1 + \sqrt{r}} = -\sqrt{r} - 1$

$$f(-\sqrt{r} - 1) = [-\sqrt{r} - 1] = \boxed{-r}$$

$\hookrightarrow \approx -r, 2$

$$f(n) = \sin n \quad g(n) = n\sqrt{1 - n^2} \quad g\left(\frac{\pi}{2}\right)$$

(3)

$$f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} = \frac{\sqrt{r}}{r}$$

(۲)

$$g\left(\frac{\sqrt{r}}{r}\right) = \frac{\sqrt{r}}{r} \sqrt{1 - \frac{1}{r}} = \frac{\sqrt{r}}{r} \sqrt{\frac{r-1}{r}} = \frac{\sqrt{r}}{r} \times \frac{\sqrt{r-1}}{\sqrt{r}} = \boxed{\frac{1}{r}}$$

الف) $f \circ g(n) = \{(5, 5), (2, 5), (4, 12), (1, 12)\}$

(4)

ب) $g \circ f(n) = \emptyset$

(۲)

ج) $f \circ f(n) = \emptyset$

د) $g \circ g(n) = \{(2, 8), (2, 7), (2, 1)\}$

$$f = \{(v,1), (v,r), (v,a), (1,v)\}$$

(5)

$$g = \{(1,r), (v,1), (a,r), (b,1)\} \quad (v,r) \in f \circ g \quad (v,1) \in g \circ f$$

(5)

$$(v,r) \in f \circ g \rightarrow f(g(v)) = r \rightarrow f(r) = r \rightarrow g(r) = v \rightarrow a = v$$

$$(v,1) \in g \circ f \rightarrow g(f(v)) = 1 \rightarrow f(v) = a \rightarrow g(a) = 1 \rightarrow b = a$$

$$(a,b) = (v,a)$$

(6)

$$f(f(n)) = f(n+1), \quad g(2n+1) = n-1$$

$$f \circ f = f(n+1) \rightarrow a(a+1) + b \rightarrow a^2n + (a+b) = f(n+1) \begin{cases} a=1, b=1 \rightarrow n+1 \\ a=1, b=-1 \rightarrow n \end{cases}$$

(5)

$$g(n) = \frac{1}{2}n - \frac{1}{2}$$

$$g \circ f(-1) \rightarrow \frac{1}{2}(2n+1) - \frac{1}{2} \rightarrow -1$$

$$g \circ f(-1) = -1$$

$$\frac{1}{2}(-2n-1) - \frac{1}{2} \rightarrow -1$$

(7)

$$f(n) = \sqrt{n+|n|} \quad \hookrightarrow D_f \subseteq \mathbb{R} \quad , \quad g(n) = \frac{1}{n^2-2n} \quad \hookrightarrow D_g \subseteq \mathbb{R} - \{0,2\}$$

(5)

$$D_{f \circ g} = \{n \in D_f, f(n) \in D_g\}$$

$$D_{g \circ f} = (0, +\infty) - \{1\}$$

$$\sqrt{n+|n|} \neq \{0,2\}$$

$$\sqrt{n+|n|} \neq 0 \rightarrow n > 0$$

$$\sqrt{n+|n|} \neq 2 \rightarrow n+|n| \neq 4 \xrightarrow{n>0} 2n \neq 4 \rightarrow n \neq 2$$

$$f(n) = \sqrt{1-n^2}$$

$$D_f \rightarrow \frac{-1+1}{-1+1}$$

$$D_f = [-1, 1] \text{ (I)}$$

$$g(n) = \sqrt{n}$$

$$D_g = [0, +\infty)$$

$$D(f+g) \text{ of } \rightarrow \{n \in D_f : f(n) \in D_{f+g}\}$$

$$D(f+g) \rightarrow [-1, 1] \cap [0, +\infty) \rightarrow [0, 1]$$

$$\sqrt{1-n^2} \in [0, 1] \rightarrow 0 \leq \sqrt{1-n^2} \leq 1 \rightarrow 0 \leq 1-n^2 \leq 1 \rightarrow 1 \geq n^2 \geq 0 \rightarrow -1 \leq n \leq 1 \text{ (II)}$$

$$D(f+g)_{\text{of}} = [-1, 1]$$

$$\text{a)} f\left(\frac{n+1}{n-2}\right) = f(n) + a \quad \frac{n+1}{n-2} = t \rightarrow n = -\frac{t-1}{t-2} = \frac{t+1}{t-2}$$

$$f(n) = \sqrt{\frac{n+1}{n-2}} + a = \frac{1}{n-2} + a = \frac{1+n-1}{n-2}$$

$$\text{b)} f\left(n + \frac{1}{n}\right) = n^{\frac{1}{2}} + \frac{1}{n^{\frac{1}{2}}} \rightarrow n + \frac{1}{n} = t \rightarrow n^{\frac{1}{2}} + \frac{1}{n^{\frac{1}{2}}} + \frac{1}{n^{\frac{1}{2}}} = t^{\frac{1}{2}}$$

$$2n^{\frac{1}{2}} + \frac{1}{n^{\frac{1}{2}}} = t^{\frac{1}{2}} - \frac{1}{n^{\frac{1}{2}}} \rightarrow f(n) = n^{\frac{1}{2}} - \frac{1}{n^{\frac{1}{2}}}$$

$$g(1) = 0 \rightarrow g(f(n)) = 0 \rightarrow f(n) = n\sqrt{n} = 1 \rightarrow n_1 = 1$$

$$g(2\sqrt{2}) = 0 \rightarrow g(f(n)) = 0 \rightarrow f(n) = n\sqrt{n} = 2\sqrt{2} \rightarrow n_2 = 2$$

$$n_2 - n_1 = 2 - 1 = 1$$