نام و نام خانوادگی ...دینا سادات رئیسی... پاسخنامه تشریحی تکلیف شماره ...۹... کلاس ...یازدهم تجربی...

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$f(x)=\begin{cases}\cot\frac{\pi x}{4} & ; x\le 1\\ \sqrt{x^2+1} & ; x>1\end{cases}$

$f\circ f(\frac{2}{3})=f(f(\frac{2}{3}))=f(\sqrt3)=\sqrt{3+1}=\sqrt4=2$

$f(\frac{2}{3})=\cot\frac{\pi\times 2}{3\times 4}=\cot\frac{\pi}{6}=\sqrt3$

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الف) $f(\frac{1}{x})=\sqrt{\frac{2x-1}{x^2}}$ $\quad g(x)=2\cos^2 x$ $\quad f\circ g(\frac{\pi}{3})=f(g(\frac{\pi}{3}))=f(\frac{1}{2})=\sqrt{\frac{2\times2-1}{4}}=\sqrt{\frac{3}{4}}=\frac{\sqrt3}{2}$

$\frac{1}{2}=\frac{1}{x}\to x=2$

$g(\frac{\pi}{3})=2\cos^2\frac{\pi}{3}=2\times\frac{1}{4}=\frac{1}{2}$

ب) $f(x)=[x]$ $\quad g(x)=\frac{x}{1-x}$ $\quad f\circ g(\sqrt2)=f(g(\sqrt2))=f(-2-\sqrt2)=[\underbrace{-2-\sqrt2}_{-3/4}]=-4$

$g(\sqrt2)=\frac{\sqrt2\times(1+\sqrt2)}{(1-\sqrt2)\times(1+\sqrt2)}=\frac{\sqrt2+2}{-1}=-2-\sqrt2$

۳

$f(x)=\sin x$

$g(x)=x\sqrt{1-x^2}$

$g\circ f(\frac{\pi}{4})=g(f(\frac{\pi}{4}))=g(\frac{\sqrt2}{2})=\frac{\sqrt2}{2}\times\sqrt{1-\frac{1}{2}}=\frac{\sqrt2}{2}\times\frac{\sqrt2}{2}=\frac{2}{4}=\frac{1}{2}$

$f(\frac{\pi}{4})=\sin\frac{\pi}{4}=\frac{\sqrt2}{2}$

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$f(x)=\{(4,5),(6,5),(8,12),(10,12)\}$

$g(x)=\{(4,6),(2,4),(6,8),(8,10)\}$

الف) $f\circ g(x)=f(g(x))=\{(4,5),(2,5),(6,12),(8,12)\}$

ب) $g\circ f(x)=g(f(x))=\varnothing$

ج) $f\circ f(x)=f(f(x))=\varnothing$

د) $g\circ g(x)=g(g(x))=\{(4,8),(2,6),(6,10)\}$

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$f=\{(2,1),(c,2),(4,5),(1,7)\}$ $\quad (4,2)\in f\circ g(x)$ $\quad (4,1)\in g\circ f$

$g=\{(1,3),(c,1),(a,3),(b,1)\}$

$(4,2)\in f\circ g \Longrightarrow 4\xrightarrow{g(x)}3\xrightarrow{f(x)}2$ $\quad a=4$ $\quad (a,b)=(4,5)$

$(4,1)\in g\circ f \Longrightarrow 4\xrightarrow{f(x)}5\xrightarrow{g(x)}1$ $\quad b=5$

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$f(f(x)) = 4x+3 \to a^2x+ab+b = 4x+3 \to a^2=4 \to a=2 \Rightarrow 2b+b=3 \to b=1$

$f(x) = ax+b \to f(f(x)) = a(ax+b)+b = a^2x+ab+b$ $\qquad f(x) = 2x+1$

$g(2x+3) = 3x-2 \qquad 2x+3=t \to x = \frac{t-3}{2} \to g(t) = \frac{3(t-3)}{2} - 2 = \frac{3t-13}{2} \to g(x) = \frac{3x-13}{2}$

$g\circ f(-1) = g(-1) = \frac{3(-1)-13}{2} = \frac{-16}{2} = -8$

$f(-1) = 2(-1)+1 = -1$

$a^2=4 \to$ اگر $\to a=-2 \to b=-3 \to f(x) = -2x-3$

$f(-1) = +2-3 = -1$

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$f(x) = \sqrt{x+|x|}$

$g(x) = \frac{1}{x^2-4x}$

$g\circ f(x) = \{x\in D_f \;;\; f(x)\in D_g\} \to$

$\left\{\begin{array}{l} x\in D_f \to x+|x|\geq 0 \to x\geq -|x| \to D_f = \mathbb{R} \\ f(x)\in D_g \to \sqrt{x+|x|}\neq 0 \to (0,+\infty) \\ D_g \Rightarrow x^2-4x\neq 0 \to x\neq 0,\ x\neq 4 \to \sqrt{x+|x|}\neq 4 \to x+|x|\neq 16 \to x\neq 8 \end{array}\right\} \to D_{g\circ f} = (0,+\infty) - \{8\}$

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$f(x) = \sqrt{1-x^2}$

$g(x) = \sqrt{x}$

دامنه $(f+g)\circ f \to \{x\in D_f \;;\; f(x)\in D_{f+g}\} \to$

$\left\{\begin{array}{l} D_f = 1-x^2\geq 0 \to 1\geq x^2 \to 1\geq x\geq -1 \\ f+g = \underbrace{\sqrt{1-x^2}}_{1\geq x\geq -1} + \underbrace{\sqrt{x}}_{x\geq 0} \to D_{f+g} \Rightarrow 1\geq x\geq 0 \\ f(x)\in D_{f+g} \to 1\geq \underbrace{\sqrt{1-x^2}\geq 0}_{\text{همواره برقرار}} \\ \to 1\geq 1-x^2 \to x^2\geq 0 \to x\geq 0 \end{array}\right\} \to D_{(f+g)\circ f} = [0,1]$

۹

الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5 \sim f(t) = 4\left(\frac{2t+1}{t-3}\right)+5 = \frac{13t-11}{t-3} \to f(x) = \frac{13x-11}{x-3}$

$\frac{3x+1}{x-2} = t \to x = -\frac{-2t-1}{t-3} = \frac{2t+1}{t-3}$

ب) $f\left(x+\frac{1}{x}\right) = x^3+\frac{1}{x^3} \to f(t) = t^3-3t \to f(x) = x^3-3x$

$\underbrace{x+\frac{1}{x}}_{t} = t$

$x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)\left(\overbrace{x^2+\frac{1}{x^2}}^{x^2+\frac{1}{x^2}+2-3}-1\right) = (t)\left((t)^2-3\right) = t^3-3t$

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$f(x) = x\sqrt{x}$

$g\circ f(x)$ در صورتی صفر می‌شود $\to f(x) = 1 \to x\sqrt{x} = 1 \to x = 1 \qquad \to g(f(1)) = 0$

$\to f(x) = 2\sqrt{2} \to x\sqrt{x} = 2\sqrt{2} \to x = 2 \qquad \to g(\underbrace{f(2)}_{2\sqrt{2}}) = 0$

$\to 2-1 = 1$