

نام و نام خانوادگی ..... دینا سادات زهیر ..... پاسخنامه تشریحی تکلیف شماره 9 کلاس ..... ریاضیات پایه

$$f(x) = \begin{cases} \cot \frac{\pi x}{x} & ; x \leq 1 \\ \sqrt{x^2 + 1} & ; x > 1 \end{cases}$$

$$f \circ f\left(\frac{1}{x}\right) = f\left(f\left(\frac{1}{x}\right)\right) = f(\sqrt{x^2 + 1}) = \sqrt{x^2 + 1} - \sqrt{x} = x$$

$$f\left(\frac{x}{x}\right) = \cot \frac{\pi x}{x \times x} = \cot \frac{\pi}{x} = \sqrt{x}$$

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$$a) f\left(\frac{1}{x}\right) = \sqrt{\frac{x^2 - 1}{x^2}} \quad g(x) = x \cos x \quad f \circ g\left(\frac{x}{x}\right) = f\left(g\left(\frac{x}{x}\right)\right) = f\left(\frac{1}{x}\right) = \sqrt{\frac{x^2 - 1}{x^2}} = \sqrt{\frac{x^2}{x^2}} = \frac{x}{x} = 1$$

$$g\left(\frac{x}{x}\right) = x \cos \frac{x}{x} = x \times \frac{1}{x} = 1$$

$$b) f(x) = \frac{x}{1-x} \quad g(x) = \frac{x}{1-x} \quad f \circ g(x) = f\left(g(x)\right) = f\left(\frac{x}{1-x}\right) = \frac{\frac{x}{1-x}}{1 - \frac{x}{1-x}} = \frac{\frac{x}{1-x}}{\frac{1-x-x}{1-x}} = \frac{\frac{x}{1-x}}{\frac{1-2x}{1-x}} = \frac{x}{1-2x}$$

2

$$f(x) = \sin x$$

$$g(x) = x \sqrt{1-x^2}$$

$$g \circ f\left(\frac{x}{x}\right) = g\left(f\left(\frac{x}{x}\right)\right) = g\left(\frac{1}{x}\right) = \frac{1}{x} \times \sqrt{1 - \frac{1}{x^2}} = \frac{1}{x} \times \frac{\sqrt{x^2 - 1}}{x} = \frac{\sqrt{x^2 - 1}}{x^2} = \frac{1}{x}$$

$$f\left(\frac{x}{x}\right) = \sin \frac{x}{x} = \frac{1}{x}$$

2

$$f(x) = \{(x, a), (y, a), (x, y), (1, y)\}$$

$$g(x) = \{(x, y), (y, x), (x, 1), (1, x)\}$$

$$a) f \circ g(x) = f(g(x)) = \{(x, a), (y, a), (y, y), (1, y)\}$$

$$b) g \circ f(x) = g(f(x)) = \emptyset$$

$$c) f \circ f(x) = f(f(x)) = \emptyset$$

$$d) g \circ g(x) = g(g(x)) = \{(x, 1), (y, y), (y, 1), (1, x)\}$$

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$$f = \{(x, 1), (x, 2), (x, a), (1, x)\}$$

$$(x, 2) \in f \circ g$$

$$(x, 1) \in g \circ f$$

$$g = \{(1, x), (x, 1), (a, x), (b, 1)\}$$

$$(x, 2) \in f \circ g \Rightarrow \begin{matrix} (x, 2) \in f \\ \downarrow \\ a = x \end{matrix} \quad \begin{matrix} (x, 2) \in f \\ \downarrow \\ a = x \end{matrix} \quad \begin{matrix} (x, 2) \in f \\ \downarrow \\ a = x \end{matrix}$$

$$(a, b) = (x, a)$$

$$(x, 1) \in g \circ f \Rightarrow \begin{matrix} (x, 1) \in g \\ \downarrow \\ b = a \end{matrix} \quad \begin{matrix} (x, 1) \in g \\ \downarrow \\ b = a \end{matrix} \quad \begin{matrix} (x, 1) \in g \\ \downarrow \\ b = a \end{matrix}$$

2

$$a_5 \rightarrow a = -2 \rightarrow b = -2 \rightarrow \text{pa. } -2n - 2$$

$\varphi(n) = \sqrt{1-n^2}$   
 $g(n) = \sqrt{n}$   
 $\text{inj. } (f+g) \circ f \rightarrow \{n \in D_f; (f(n)) \in D_{f+g}\} \rightarrow$

$$\left\{ \begin{array}{l} D_f = \{ -n^2 \geq 0 \rightarrow 1 \geq n^2 \rightarrow 1 \geq n \geq -1 \\ f \circ g = \sqrt{1-n^2} + \sqrt{n} \rightarrow \varphi \circ g \rightarrow 1 \geq n \geq -1 \rightarrow n \geq 0 \\ \rightarrow 1 \geq n \geq 0 \rightarrow n \geq 0 \end{array} \right\} \rightarrow D_{(f+g) \circ f} = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$$

$f(n) \in D_{f+g} \rightarrow 1 \geq \sqrt{1-n^2} \geq 0 \rightarrow n^2 \leq 1$   
 $\rightarrow 1 \geq 1-n^2 \rightarrow n^2 \geq 0 \rightarrow n \geq 0$

$$q_r \frac{1}{n} = t \quad \text{---} \quad t$$

$$q_r \frac{1}{n} = \left( x_r \frac{1}{n} \right) \times \left( h_r \frac{1}{n} - 1 \right) = (+) (t)^r - r = t^r - r +$$

$f(x) = x\sqrt{x} \rightarrow g(f(x)) = 0$   
 $g(f(x)) \xrightarrow{f(x)=1} f(x) = 1 \rightarrow x\sqrt{x} = 1 \rightarrow x = 1 \rightarrow x - 1 = 0$   
 $f(x) = x\sqrt{x} \rightarrow x\sqrt{x} = \sqrt{x} \rightarrow x = 1 \rightarrow x - 1 = 0$   
 $g(f(x)) = 0$