

عملی کار

$$f(x) = \begin{cases} \cot + \frac{\pi x}{L} & x \leq 1 \\ \frac{x^2}{\sqrt{x^2+1}} & x > 1 \end{cases} \rightarrow f\left(\frac{L}{\pi}\right) \xrightarrow{\frac{L}{\pi} < 1} \cot + \pi x \frac{L}{\pi} \leftarrow \text{داده}$$

$$\rightarrow \cot + \frac{\pi}{4} = \sqrt{3} \rightarrow f(\sqrt{3}) \xrightarrow{\sqrt{3} > 1} \frac{(\sqrt{3})^2}{\sqrt{(\sqrt{3})^2+1}} = \sqrt{2}$$

$$= \boxed{2}$$

سوال ←

$$f \circ g\left(\frac{\pi}{\sqrt{2}}\right) \leftarrow g(x) = x \cos^2 x \quad \text{و} \quad f\left(\frac{1}{x}\right) = \sqrt{\frac{x^2-1}{x^2}} \quad (\text{الف})$$

$$\rightarrow x = \frac{\pi}{\sqrt{2}} \rightarrow x\left(\frac{1}{x}\right)^2 = \frac{1}{x} \rightarrow \frac{1}{x} = \frac{1}{\frac{\pi}{\sqrt{2}}} \Rightarrow x = \frac{\pi}{\sqrt{2}}$$

$$\rightarrow \sqrt{\frac{x^2-1}{x^2}} = \sqrt{\frac{(\frac{\pi}{\sqrt{2}})^2-1}{(\frac{\pi}{\sqrt{2}})^2}} = \sqrt{\frac{3}{2}} = \boxed{\frac{\sqrt{3}}{\sqrt{2}}}$$

ب.

$$f \circ g(\sqrt{2}) \quad g(x) = \frac{x}{1-x} = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = -\sqrt{2}-2$$

$$\rightarrow f(-2-\sqrt{2}) \rightarrow [-2-\sqrt{2}] = \boxed{-2}$$

سوال ←

$$g \circ f\left(\frac{\pi}{2}\right) \quad g(x) = x\sqrt{1-x^2} \quad f(x) = \sin x$$

$$\rightarrow f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} = \sqrt{\frac{1}{2}} \quad g\left(\sqrt{\frac{1}{2}}\right) = \sqrt{\frac{1}{2}} \sqrt{1-\left(\sqrt{\frac{1}{2}}\right)^2}$$

$$\rightarrow \sqrt{\frac{1}{2}} \times \sqrt{\frac{1}{2}} \rightarrow \sqrt{\frac{1}{2}} \times \sqrt{\frac{1}{2}} = \boxed{\frac{1}{2}}$$

سوال ←

$$\text{الف)} f \circ g(x) = \{(1,0)(2,0)(4,1)(8,1)\}$$

ب) $g \circ f(x) = \emptyset$

ج) $f \circ f(x) = \emptyset$

د) $g \circ g = \{(2,4)(4,1)(6,1)\}$

$$f(2,1)(3,2)(5,0)(1,7)$$

سوال ۱

$$g = \{(1,2)(3,1)(a,3)(b,1)\}$$

$$(2,x) \in f \circ g \rightarrow f(x) = 2 \rightarrow x = 3 \rightarrow g(x) = 3 \rightarrow x = a \rightarrow a = 2$$

$$(2,1) \in g \circ f \rightarrow g(x) = 1 \rightarrow x = 3 \rightarrow f(x) = 3 \rightarrow x = b \rightarrow f(x) = b \rightarrow b = 0$$

$$\rightarrow (2,0)$$

$$f(f(x)) = x+3 \rightarrow a(ax+b)+b = x+3 \leftarrow \text{سوال ۲}$$

$$\rightarrow ax+ab+b = x+3 \rightarrow a^2 = 1 \rightarrow a = -1 \rightarrow b = 3$$

$$g(x+3) = x-2 \rightarrow a(x+3)+b = x-2 \rightarrow ax+3a+b = x-2$$

$$\rightarrow a = 1, b = -5 \rightarrow g(x) = x - \frac{1}{2} \rightarrow -\frac{5}{2} - \frac{1}{2}$$

$$f(x) = x+1 \rightarrow -2+1 = -1$$

$$\boxed{-1} = -\frac{1}{2}$$

$$g(x) = \frac{1}{x^2-1} \quad f(x) = \sqrt{x+1} \rightarrow \mathbb{R} \leftarrow \text{سوال ۳}$$

$$g \circ f \rightarrow \text{نقطه } f \rightarrow \text{Domain} = \{x \in \mathbb{R} \mid f(x) \in \text{Domain of } g\}$$

$$\rightarrow \text{Domain} = \sqrt{x+1} \neq \{0,1\}$$

$$\rightarrow \text{Domain} = \mathbb{R} - \{0,1\}$$

$$\text{Domain} = [-1,1] \rightarrow 1-x^2 \geq 0 \rightarrow x^2 \leq 1 \rightarrow -1 \leq x \leq 1 \leftarrow \text{سوال ۴}$$

$$\text{Range of } f+g = \{x \in \mathbb{R} \mid f(x) \in \text{Domain of } f+g\} = \{x \in [-1,1], 0 \leq \sqrt{1-x^2} \leq 1\}$$

$$\rightarrow 0 \leq \sqrt{1-x^2} \leq 1 \rightarrow \text{Range of } f+g = [-1,1]$$

Scanned with

$$a) f\left(\frac{x+1}{x-2}\right) \leq (x+2)$$

سوال 9 ←

$$\frac{x+1}{x-2} = t \rightarrow t(x-2) = x+1 \rightarrow tx - 2t = x+1 \rightarrow x(t-1) = 2t+1 \rightarrow x = \frac{2t+1}{t-1}$$

$$\rightarrow x \leq \frac{2t+1}{t-1} \rightarrow \left(\frac{2t+1}{t-1}\right) \leq \frac{2t+1}{t-1} \rightarrow \frac{2t+1}{t-1} \leq \frac{2t+1}{t-1} \rightarrow \frac{2t+1}{t-1} \leq \frac{2t+1}{t-1}$$

$$b) f\left(x + \frac{1}{x}\right) = \frac{x^3 + 1}{x^2} \rightarrow \left(x + \frac{1}{x}\right)^3 = x^3 + \frac{3}{x} + \frac{3}{x} + \frac{1}{x^3} = x^3 + \frac{6}{x} + \frac{1}{x^3}$$

$$x + \frac{1}{x} = t \rightarrow t^3 = x^3 + \frac{6}{x} + \frac{1}{x^3}$$

$$\rightarrow \boxed{f(t) \leq x^3 - 6x}$$

$$g(x) = (x+1)(x-2\sqrt{x}) = x^2 - (2\sqrt{x}+1)x + 2\sqrt{x} \leftarrow \text{سوال 10}$$

$$f(x) = x\sqrt{x} \quad (x\sqrt{x})^2 - (2\sqrt{x}+1)(x\sqrt{x}) + 2\sqrt{x}$$

$$\rightarrow (x\sqrt{x} - 2\sqrt{x})(x\sqrt{x} - 1) \rightarrow x = 2, x = 1$$

$$\rightarrow 2 \leq 1 \quad \boxed{1}$$