عسل طاهری

سوال ۱ ←

$f(x) = \begin{cases} \cot \frac{\pi x}{4} & x \le 1 \\ \sqrt{x^2+1} & x > 1 \end{cases} \to f(\frac{2}{3}) \xrightarrow{\frac{2}{3}<1} \cot \frac{\pi \times \frac{2}{3}}{4}$

$\to \cot \frac{\pi}{6} = \sqrt{3} \longrightarrow f(\sqrt{3}) \xrightarrow{\sqrt{3}>1} \sqrt{(\sqrt{3})^2+1} = \sqrt{4}$

$= \boxed{2}$

(۲)

سوال ۲ ←

الف) $f(\frac{1}{x}) = \sqrt{\frac{3x-1}{x^2}}$ و $g(x) = 2\cos^2 x$ ← $fog(\frac{\pi}{3})$

$\to 2\cos^2\frac{\pi}{3} \to 2(\frac{1}{2})^2 = \frac{1}{2} \longrightarrow \frac{1}{x} = \frac{1}{2} \Rightarrow x = 2$

$\longrightarrow \sqrt{\frac{2(2)-1}{(2)^2}} = \sqrt{\frac{3}{4}} = \boxed{\frac{\sqrt{3}}{2}}$

(۲)

ب) $fog(\sqrt{2})$ $\quad g(x) = \frac{x}{1-x} = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = -\sqrt{2}-2$

$\to f(-2-\sqrt{2}) \to [-2-\sqrt{2}] = \boxed{-4}$

سوال ۳ ←

$gof(\frac{\pi}{4})$ $\quad g(x) = x\sqrt{1-x^2}$ $\quad f(x) = \sin x$

$\to f(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\sqrt{1-(\frac{\sqrt{2}}{2})^2}$

$\to \frac{\sqrt{2}}{2} \times \sqrt{\frac{1}{2}} \to \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \boxed{\frac{1}{2}}$

(۲)

سوال ۴ ←

الف) $fog(x) = \{(1,5)(2,5)(6,14)(8,12)\}$

ب) $gof(x) = \varnothing$

ج) $fof(x) = \varnothing$

د) $gog = \{(2,6)(6,10)(5,8)\}$

(۲)

سوال ۵ ←

$f(2,1)(3,2)(4,5)(1,7)$

$g=\{(1,2)(3,1)(a,3)(b,1)\}$

$(4,2)\in f\circ g \longrightarrow f(x)=2 \to x=3 \longrightarrow g(x)=3 \to x=a \to a=4$

$(4,1)\in g\circ f \longrightarrow g(x)=1 \to x=3 \to f(x)=3 \to$ ✗

$\to x=b \to f(4)=b \to b=5$

$\longrightarrow (4,5)$

(۴)

سوال ۶ ←

$f(f(x))=4x+3 \longrightarrow a(ax+b)+b=4x+3$

$\longrightarrow a^2x+ab+b=4x+3 \longrightarrow a^2=4$ ، $a=-2 \to b=-3$ ، $a=2 \longrightarrow b=1$

$g(2x+3)=3x-2 \longrightarrow a(2x+3)+b=3x-2 \to 2ax+3a+b=3x-2$

$\to a=\frac{3}{2}, b=-\frac{13}{2} \to g(x)=\frac{3}{2}x-\frac{13}{2} \longrightarrow -\frac{3}{2}-\frac{13}{2}$

$f(x)=2x+1 \to -2+1=\boxed{-1}$

$\boxed{-8}=-\frac{16}{2}$

(۴)

سوال ۷ ←

$g(x)=\frac{1}{x^2-4x} \to x(x-4) \to R-\{0,4\}$ $\quad f(x)=\sqrt{x+|x|} \to D_f=R$

$g\circ f \longrightarrow$ دامنه $\to D_{g\circ f}=\{x\in D_f \mid f(x)\in D_g\}$

$\longrightarrow D_{g\circ f}=\{x\in R \mid \sqrt{x+|x|}\neq\{0,4\}\}$

$\to D_{g\circ f}=R-\{0,4\}$

$\sqrt{x+|x|}\neq 0 \to x>0$

$\sqrt{x+|x|}\neq 4 \to x\neq 8$ $\to D_{g\circ f}=(0,+\infty)-\{8\}$

(۱)

سوال ۸ ←

$D_f=[-1,1] \longrightarrow 1-x^2\geq 0 \longrightarrow x^2\leq 1 \longrightarrow -1\leq x\leq 1$

$D_{(f+g)\circ f}=\{x\in D_f, f(x)\in D_{f+g}\}=\{x\in[-1,1], 0\leq\sqrt{1-x^2}\leq 1\}$

$\longrightarrow 0\leq\sqrt{1-x^2}\leq 1 \xrightarrow{\text{همیشه صدق می‌کند}} D_{(f+g)\circ f}=[-1,1]$

(۴)

سوال ۹ ←

الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5$

$\frac{3x+1}{x-2} = t \to tx - 2t = 3x+1$ ۲

$tx - 3x = 2t+1 \to x(t-3) = 2t+1$

$\to x = \frac{2t+1}{t-3} \to 4\left(\frac{2t+1}{t-3}\right) = \frac{8t+4}{t-3} + 5 \to \frac{8t+4+5t-15}{t-3}$

$\to \frac{13x-11}{x-3}$

ب) $f\left(x+\frac{1}{x}\right) = x^3+\frac{1}{x^3} \to \left(x+\frac{1}{x}\right)^3 - 3x \times \frac{1}{x}\left(x+\frac{1}{x}\right)$

$x+\frac{1}{x} = t \to t^3 - 3(t) = t^3-3t$

$\to \boxed{f(x) = x^3 - 3x}$

سوال ←

$g(x) = (x+1)(x-2\sqrt{2}) = x^2 - (2\sqrt{2}+1)x + 2\sqrt{2}$

$f(x) = x\sqrt{x}$ $\quad (x\sqrt{x})^2 - (2\sqrt{2}+1)(x\sqrt{x}) + 2\sqrt{2}$

$\to (x\sqrt{x} - 2\sqrt{2})(x\sqrt{x} - 1) \to x=2 , x=1$ ۲

$\to 2 - 1 = \boxed{1}$