

زهره آصفی

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گروه ریاضی

$$f(x) = \begin{cases} \cot \frac{\pi}{x} & x \leq 1 \\ \sqrt{x^2 + 1} & x > 1 \end{cases}$$

$$f\left(\frac{x}{y}\right) \rightarrow \cot \frac{\pi}{\frac{x}{y}} \rightarrow \frac{\pi}{\frac{y}{x}} \rightarrow \sqrt{3} \quad (1)$$

$$f(\sqrt{3}) \rightarrow \sqrt{(\sqrt{3})^2 + 1} = 2$$

$$f \circ g\left(\frac{1}{x}\right) \rightarrow g\left(\frac{1}{x}\right) \rightarrow \frac{1}{\cos^2\left(\frac{\pi}{\frac{1}{x}}\right)} = \frac{1}{\left(\frac{1}{x}\right)^2} \quad f\left(\frac{1}{x}\right) = f\left(\frac{1}{x}\right) \rightarrow$$

$$f \circ g\left(\frac{1}{x}\right) = \sqrt{\frac{1}{x^2} + 1} \quad \leftarrow \sqrt{\frac{1+x^2}{x^2}} \xrightarrow{x \rightarrow \frac{1}{x}} f\left(\frac{1}{x}\right) = \sqrt{\frac{x-1}{x}}$$

$$f \circ g(\sqrt{x}) \rightarrow g(\sqrt{x}) = \frac{\sqrt{x}}{1-\sqrt{x}} \rightarrow \frac{\sqrt{x}}{1-\sqrt{x}} \times \frac{1+\sqrt{x}}{1+\sqrt{x}} \rightarrow \frac{\sqrt{x}+x}{1-x} = -\sqrt{x}-x$$

$$f(-\sqrt{x}-x) = \left[\frac{-\sqrt{x}-x}{-x^2} \right] = -x$$

$$f(x) = \sin x \quad g(x) = x \sqrt{1-x^2} \quad g\left(\frac{1}{x}\right)$$

$$f\left(\frac{1}{x}\right) = \sin \frac{1}{x} = \frac{\sqrt{x}}{x} \quad g\left(\frac{\sqrt{x}}{x}\right) = \frac{\sqrt{x}}{x} \sqrt{1 - \left(\frac{\sqrt{x}}{x}\right)^2} \rightarrow \left[\frac{1}{x} \right]$$

$$1) f \circ g(x) \rightarrow \{(1,0), (2,0), (4,1), (1,1)\} \quad 2) g \circ f(x) = \emptyset$$

$$3) f \circ f(x) = \emptyset \quad 4) g \circ g(x) \rightarrow \{(1,1), (2,1), (4,1)\}$$

$$f = \{(1,1), (2,2), (4,0), (1,0)\} \quad g = \{(1,1), (2,1), (4,1), (1,1)\}$$

$$f, g \in f \circ g \quad (1,0) \in f \circ f$$

$$f(g(1)) = 1$$

$$g(1) = 1 \rightarrow 1 = f$$

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$$f(f(a)) = \varepsilon a + 1$$

$$g(x+y) = \varepsilon x + y$$

$$g \circ f(-1) = \varepsilon$$

(5)

$$f \circ f = \varepsilon a + 1$$

$$\hookrightarrow g(a) = \frac{\varepsilon}{1} x \rightarrow -\frac{1}{\varepsilon}$$

$$\hookrightarrow a(a+b) + b \rightarrow a^2 + (a+b) = \varepsilon a + 1 \rightarrow a = 1 \quad b = 1 \rightarrow \varepsilon a + 1$$

$$a = -1 \quad b = -1 \rightarrow \varepsilon a + 1$$

$$g \circ f(-1) = \frac{\varepsilon}{1} (\varepsilon a + 1) - \frac{1}{\varepsilon} \rightarrow -1$$

$$\frac{\varepsilon}{1} (-\varepsilon a - 1) - \frac{1}{\varepsilon} \rightarrow -1$$

$$f(-1) = 1$$

(6)

$$f(x) = \sqrt{x+1}$$

$$g(x) = \frac{1}{x^2 - \varepsilon x}$$

$$D_{g \circ f}$$

$$\frac{1}{(\sqrt{x+1})(\sqrt{x+1} - \varepsilon)} = \frac{1}{\cancel{\sqrt{x+1}} - \sqrt{x+1}} \rightarrow D_{g \circ f} = (0, +\infty) - \{1\}$$

(7)

$$f(x) = \sqrt{1-x^2}$$

$$g(x) = \sqrt{x}$$

$$(f \circ g) \text{ of } = ?$$

$$\sqrt{x} + \sqrt{1-x^2} \rightarrow \sqrt{x^2} + \sqrt{1-x^2}$$

$$\frac{-1}{-1+1} = \frac{1}{-1+1}$$

$$\rightarrow [-1, 1]$$

$$\text{a)} f\left(\frac{x+1}{x-1}\right) = \varepsilon a + 1$$

$$\frac{x+1}{x-1} = t \rightarrow x = \frac{t-1}{t-1} = \frac{t+1}{t-1}$$

(8)

$$f(x) = f\left(\frac{x+1}{x-1}\right) + 1$$

$$\frac{1+x+1}{x-1} + 1 = \frac{1+x+1+2x-2}{x-1}$$

$$\frac{1+x-1}{x-1}$$

$$\text{b)} f\left(\frac{a+\frac{1}{a}}{t}\right) = x^2 + \frac{1}{x^2} \rightarrow x^2 + \frac{1}{x^2} + \varepsilon\left(\frac{a+\frac{1}{a}}{t}\right) = t^2$$

$$\rightarrow x^2 + \frac{1}{x^2} + \varepsilon\left(\frac{a+\frac{1}{a}}{t}\right) = t^2$$

$$\hookrightarrow x^2 + \frac{1}{x^2} + \varepsilon\left(\frac{a+\frac{1}{a}}{t}\right) = t^2$$

$$f(x) = x^2 - \varepsilon x$$

$$g(1) = 0$$

$$g(f(a)) = 0 \rightarrow f(a) = x\sqrt{a} = 1 \rightarrow a = 1$$

(9)

$$g(\sqrt{1}) = 0$$

$$g(f(a)) = f(a) = x\sqrt{a} = \sqrt{1} \rightarrow a = 1$$

$$1-1 = 0$$