۱۹٫۷۵

شهزاد شوشتری ۹

$f\circ f(\frac{2}{3}) = f(f(\frac{2}{3}))$ $\cot\frac{\pi x}{4} \to \cot\frac{\pi\frac{2}{3}}{4} \to \cot\frac{\pi}{6} = \to \sqrt{3}$ ۱)

$f(\sqrt{3}) = \sqrt{3+1} = \sqrt{4} = 2$ ۲

$f\circ f(\frac{2}{3}) = \boxed{2}$

$f(\frac{1}{x}) = \sqrt{\frac{kx-1}{x^2}}$ $g(x) = 2\cos^2 x \to g(\frac{\pi}{3}) = 2\cos^2\frac{\pi}{3} = \frac{1}{2}$ $\frac{1}{2} = \frac{1}{x} \to x = 2$ $\sqrt{\frac{4-1}{4}} = \sqrt{\frac{3}{4}} = \boxed{\frac{\sqrt{3}}{2}}$ ۲) الف) ۲

$f\circ g(\sqrt{2}) = f(g(\sqrt{2})) \to g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \to f(\frac{\sqrt{2}}{1-\sqrt{2}}) = [\frac{\sqrt{2}}{1-\sqrt{2}}] = \boxed{-4}$

۱٫۴ ۱٫۴

$g\circ f(\frac{\pi}{4}) = g(f(\frac{\pi}{4}))$ ب)

$\frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \boxed{\frac{1}{2}}$ $f(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\sqrt{1-\frac{2}{4}} = \frac{\sqrt{2}}{2}\times\sqrt{\frac{1}{2}} =$ ۲ ۳)

الف) $f\circ g = \{(4,5)(3,5)(6,12)(8,12)\}$

ب) $g\circ f = \emptyset$ ۴)

ج) $f\circ f = \emptyset$ ۲

د) $g\circ g = \{(4,8)(3,9)(6,10)\}$

$(4,2) \in f\circ g$ $g = \{(1,2)(3,1)(a,3)(b,1)\}$ $f = \{(2,1)(3,2)(4,5)(1,7)\}$ ۵)
$(4,1) \in g\circ f$

$g\{(a,3)\} \to a = 5$ ۲
$\to (a,b) = (5,5)$
$g\{(b,1)\} \to b = 5$

شهرزاد شوشتری

۶)

$f\circ f = 4x + 3 \qquad f = ax + b \to f\circ f = a(ax+b)+b \to a^2x + ab + b = 4x + 3$

$a = \pm 2 \to b = 1, -3$

$g(2x+3) = 3x - 2 \to 2x + 3 = t \to x = \frac{t-3}{2} \to 3\left(\frac{t-3}{2}\right) - 2$

$\frac{3t - 9 - 4}{2} = \frac{3t - 13}{2} \to g(x) = \frac{3x - 13}{2}$

$f = 2x + 1 = -2 + 1 = -1 \to g(-1) = \frac{-3 - 13}{2} = \frac{-16}{2} = -8$

۲

---

۷)

$g(x) = \frac{1}{x^2 - 4x} \qquad f(x) = \sqrt{x + |x|}$

$x \in D_f,\ f(x) \in D_g \to D_f \Rightarrow x + |x| \geq 0 \quad |x| \geq -x \to R$

$D_g \to x^2 - 4x \neq 0 \to x(x-4) \neq 0 \to x \neq 0, 4$

$f(x) \neq 0, 4 \to \sqrt{x + |x|} \neq 0 \to f(x) \neq (-\infty, 0]$

$\sqrt{x + |x|} \neq 4 \to x + |x| \neq 16 \to x \neq 8$

$\xrightarrow{\cap} D_{g\circ f} = (0, +\infty) - \{8\}$

۲

---

۸)

۱,۷۵

$x \in D_f,\ f(x) \in D_{(f+g)} \to D_f \to 1 - x^2 \geq 0 \quad x^2 \leq 1 \to -1 \leq x \leq 1$

$D_g \to |x| \geq 0 \qquad D_{(g+f)} \to [0, 1] \qquad f_x \in [0,1] : 0 \leq \sqrt{1 - x^2} \leq 1$

$\xrightarrow{\cap} D_{((f+g)\circ f)} = \cancel{[0,1]}$ $[-1,1]$

$0 \leq 1 - x^2 \leq 1 \to -1 \leq -x^2 \leq 0 \to 0 \leq x^2 \leq 1 \to 0 \leq x \leq 1 \to [0, 1]$

$\hookrightarrow x^2 \leq 1, \checkmark$

شهرزاد شوشتری

۹)

الف)

$f\left(\frac{3x+1}{x-2}\right)=4x+5 \qquad \frac{3x+1}{x-2}=t \to x=\frac{-(-2t-1)}{t-3}=\frac{2t+1}{t-3} \to$

$f\left(\frac{2t+1}{t-3}\right)+5 \Rightarrow \frac{8t+4+5t-15}{t-3}=\frac{13t-11}{t-3} \to \boxed{f(x)=\frac{13x-11}{x-3}}$

۳

ب)

$f\left(x+\frac{1}{x}\right)=x^3+\frac{1}{x^3} \to x+\frac{1}{x}=t \qquad x^3+\frac{1}{x^3}+3\underbrace{\left(x+\frac{1}{x}\right)}_{t}=t^3$

$x^3+\frac{1}{x^3}=t^3-3t \to f(x)=x^3-3x$

---

۱۰)

$x\sqrt{x}=1 \to x=1$

$x\sqrt{x}=2 \to x=2 \to 2-1=1$

۲