نام و نام خانوادگی : آرمین عظیمی — تمرین : ۹ — کلاس : یازدهم ریاضی

$$f(x)\begin{cases}\cot\frac{\pi x}{4} & x\le 1\\ \sqrt{x^2+1} & x>1\end{cases}$$

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$f(f(\frac{2}{3}))$ $\xrightarrow{\frac{2}{3}<1}$ $f(\frac{2}{3}) = \cot\frac{2\pi}{12} = \cot\frac{\pi}{6} = \sqrt{3} \rightarrow f(\sqrt{3}) : \underline{\sqrt{3}>1}$

$f(\sqrt{3}) = \sqrt{(\sqrt{3})^2+1} = 2$

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الف) $f(\frac{1}{x}) = \sqrt{\frac{2x-1}{x^2}} \rightarrow f(x) = \sqrt{\frac{2(\frac{1}{x})-1}{(\frac{1}{x})^2}} = \sqrt{\frac{\frac{2}{x}-1}{\frac{1}{x^2}}} = \sqrt{\frac{\frac{2-x}{x}}{\frac{1}{x^2}}} = \sqrt{-x^2+2x} = f(x)$

$g(x) = 2\cos^2 x$

$\Rightarrow f(g(\frac{\pi}{3})) : \rightarrow g(\frac{\pi}{3}) = 2\cos^2\frac{\pi}{3} = 2(\frac{1}{2})^2 = \frac{1}{2}$

$f(\frac{1}{2}) = \sqrt{-(\frac{1}{2})^2+2(\frac{1}{2})} = \sqrt{-\frac{1}{4}+1} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

ب) $f(x) = [x]$, $g(x) = \frac{x}{1-x}$ $\Rightarrow f(g(\sqrt{2})) : g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{-1} = -\sqrt{2}-2$

$f(-\sqrt{2}-2) = [-\sqrt{2}-2] = [-3/...] = -4$

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$f(x) = \sin x$, $g(x) = x\sqrt{1-x^2}$ $\Rightarrow g(f(\frac{\pi}{4})) : \rightarrow f(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\left(\sqrt{1-(\frac{\sqrt{2}}{2})^2}\right) = \frac{\sqrt{2}}{2}\left(\sqrt{\frac{1}{2}}\right) =$

$\frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$

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$f(x) = \{(4,5)(6,5)(8,12)(10,12)\}$

$g(x) = \{(4,6)(2,4)(6,8)(8,10)\}$

ب) $gof(x) = g(f(x))$: x: $4 \xrightarrow{f(x)} 5 \xrightarrow{g(x)} \times$, $6 \rightarrow 5 \rightarrow \times$, $8 \rightarrow 12 \rightarrow \times$, $10 \rightarrow 12 \rightarrow \times$; $\varnothing$

ج) $fof(x) = f(f(x))$: x: $4 \xrightarrow{f(x)} 5 \xrightarrow{f(x)} \times$, $6 \rightarrow 5 \rightarrow \times$, $8 \rightarrow 12 \rightarrow \times$, $10 \rightarrow 12 \rightarrow \times$; $\varnothing$

الف) $fog(x) = f(g(x))$: x: $4 \xrightarrow{g(x)} 6 \xrightarrow{f(x)} 5 \rightarrow (4,5)$, $2 \rightarrow 4 \rightarrow 5 \rightarrow (2,5)$, $6 \rightarrow 8 \rightarrow 12 \rightarrow (6,12)$, $8 \rightarrow 10 \rightarrow 12 \rightarrow (8,12)$

د) $g(g(x))$: x: $4 \xrightarrow{g(x)} 6 \xrightarrow{g(x)} 8 \rightarrow (4,8)$, $2 \rightarrow 4 \rightarrow 6 \rightarrow (2,6)$, $6 \rightarrow 8 \rightarrow 10 \rightarrow (6,10)$, $8 \rightarrow 10 \rightarrow \times$

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$f = \{(2,1)(3,2)(4,5)(1,6)\}$ $\qquad (a,b) = \boxed{(4,5)}$

$g = \{(1,2)(3,1)(a,3)(b,1)\}$ $\qquad a = 4$

$(4,2) \in f(g(x)) \Rightarrow f(g(x)) : \begin{cases}x\in D_g\\ g(x)\in D_f\end{cases} \Rightarrow \begin{cases}4\in D_g\\ 2\in R_f \rightarrow g(x)=3\end{cases}$ $\quad 4\in D_g$ $(4,3)$

$(4,1) \in g(f(x)) \Rightarrow \begin{cases}4\in D_f\\ f(x)\in D_g\end{cases} \Rightarrow \begin{cases}4\in D_f\\ 1\in R_g \rightarrow f(x)=b\end{cases}$ $4\in D_f$ $f(4)=b=5 \rightarrow b=5$

$f(f(x)) = 4x+3 \rightsquigarrow f(x) = ax+b \to f(ax+b) = a(ax+b)+b = 4x+3$

$a^2x + ab + b = 4x+3$

$\Rightarrow \begin{cases} a^2 = 4 \\ ab+b = 3 \end{cases}$ → $a=2 \to 3b = 3 \to b=1$, $f(x) = 2x+1$; → $a=-2 \to -b = 3 \to b=-3$, $f(x) = -2x-3$ 6

$g(\underbrace{2x+3}_{t}) = 3x-2$

$2x+3 = t$, $x = \frac{t-3}{2}$; $g(t) = 3\left(\frac{t-3}{2}\right) - 2$

$g(x) = 3\left(\frac{x-3}{2}\right) - 2 = \frac{3x-9-4}{2} = \underbrace{\frac{3x-13}{2} = g(x)}$

$\Rightarrow g(f(-1)) \rightsquigarrow f(-1) \begin{cases} 2x+1 = -2+1 = -1 \\ -2x-3 = 2-3 = -1 \end{cases} = -1 \to g(-1) = \frac{3(-1)-13}{2} = \frac{-16}{2} = -8$

$f(x) = \sqrt{x+|x|} \to D_f : x+|x| \geq 0$ $\begin{matrix} + ✓ \\ - ✓ \\ 0 ✓ \end{matrix}$ $\to D_f = \mathbb{R}$

$g(x) = \frac{1}{x^2-4x} \to D_g : x^2-4x \neq 0$, $x(x-4) \neq 0 \to x\neq 0, x \neq 4 \to D_g : \mathbb{R} - \{0, 4\}$

$D_{gof} : \to \begin{cases} x \in D_f \\ f(x) \in D_g \end{cases} \Rightarrow ① \; x \in \mathbb{R}$

$② \; \sqrt{x+|x|} \neq 0 \to x+|x| \neq 0 \to x = (0, +\infty)$

$\sqrt{x+|x|} \neq 4 \to x+|x| \neq 16 \to x \neq 8$ $\Big\} \; x = (0, +\infty) - \{8\}$

$①\cap② \Rightarrow \boxed{D_{gof} : (0, +\infty) - \{8\}} = \mathbb{R}^+ - \{8\}$ 7

$f(x) = \sqrt{1-x^2} \to D_f : 1-x^2 \geq 0$, $x^2 \leq 1$, $-1 \leq x \leq 1$

$g(x) = \sqrt{x} \to D_g : x \geq 0$

$\Rightarrow D_{f+g} = D_f \cap D_g \Rightarrow D_{f+g} : [0, 1]$

$D_{(f+g)of} : \begin{cases} x \in D_f \\ f(x) \in D_{f+g} \end{cases} \Rightarrow ① \; -1 \leq x \leq 1$

$② \; \underbrace{0 \leq}_{(I)} \underbrace{\sqrt{1-x^2} \leq 1}_{(II)} \to$

$(I) \; \sqrt{1-x^2} \geq 0 \xrightarrow{\text{توان ۲}} 1-x^2 \geq 0 \to x^2 \leq 1$, $-1 \leq x \leq 1$

$(II) \; \sqrt{1-x^2} \leq 1 \xrightarrow{2} 1-x^2 \leq 1$, $x^2 \geq 0 \;\; \mathbb{R}$ $\Big\} -1 \leq x \leq 1$

$①\cap② \Rightarrow D_{(f+g)of} : \boxed{[-1, 1]}$ 8

الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5 \to \frac{3x+1}{x-2} = t \to x = -\frac{-2t-1}{t-3} = \frac{2t+1}{t-3}$

$\to f(t) = 4\left(\frac{2t+1}{t-3}\right) + 5 \to f(t) = \frac{8t+4+5t-15}{t-3} \to \boxed{f(x) = \frac{13x-11}{x-3}}$

ب) $f\left(x+\frac{1}{x}\right) = x^3 + \frac{1}{x^3} \to$ $\left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x + 3\frac{1}{x}$

$x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right) \Rightarrow f(t) = t^3 - 3t$

$x^3 + \frac{1}{x^3} = t^3 - 3t$; $\boxed{f(x) = x^3 - 3x}$ 9

ریشه‌های g : $2\sqrt{2}, 1$

$f(x) = x\sqrt{x}$

اختلاف ریشه‌ها

$g(f(x)) : ? \Rightarrow$ ریشه‌های $g(x) = 2\sqrt{2}, 1 \to$ $f(x) = 2\sqrt{2} \to x\sqrt{x} = 2\sqrt{2} \to x = 2$; $f(x) = 1 \to x\sqrt{x} = 1 \to x = 1$ $\Big\}$ ریشه‌های $gof(x)$ 10

اختلاف = $2-1 = 1$