کلاس: یازدهم [illegible] — تعداد: ۹ — ۲۰ — نام و نام خانوادگی: آرین عظیمی

۱ (۲)

$$f(x)\begin{cases}\cot\frac{\pi x}{4} & x\le 1\\ \sqrt{x^2+1} & x>1\end{cases}$$

$f(f(\frac{2}{3}))\quad \underline{\frac{2}{3}<1}\quad f(\frac{2}{3})=\cot\frac{2\pi}{12}=\cot\frac{\pi}{6}=\sqrt{3}\rightarrow f(\sqrt{3}):\ \underline{\sqrt{3}>1}$

$f(\sqrt{3})=\sqrt{(\sqrt{3})^2+1}=2$

۲ (۲)

الف) $f(\frac{1}{x})=\sqrt{\frac{2x-1}{x^2}}\rightarrow f(x)=\sqrt{\frac{2(\frac{1}{x})-1}{(\frac{1}{x})^2}}=\sqrt{\frac{\frac{2}{x}-1}{\frac{1}{x^2}}}=\sqrt{\frac{\frac{2-x}{x}}{\frac{1}{x^2}}}=\sqrt{-x^2+2x}=f(x)$

$g(x)=2\cos^2 x$

$\Rightarrow f(g(\frac{\pi}{3})):\rightarrow g(\frac{\pi}{3})=2\cos^2\frac{\pi}{3}=2(\frac{1}{2})^2=\frac{1}{2}$

$f(\frac{1}{2})=\sqrt{-(\frac{1}{2})^2+2(\frac{1}{2})}=\sqrt{-\frac{1}{4}+1}=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$

ب) $f(x)=[x],\ g(x)=\frac{x}{1-x}\Big\}\Rightarrow f(g(\sqrt{2})):\ g(\sqrt{2})=\frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{1+\sqrt{2}}{1+\sqrt{2}}=\frac{\sqrt{2}+2}{-1}=-\sqrt{2}-2$

$f(-\sqrt{2}-2)=[-\sqrt{2}-2]=[-3/\dots]=-4$

۳ (۲)

$f(x)=\sin x,\ g(x)=x\sqrt{1-x^2}\Big\}\Rightarrow g(f(\frac{\pi}{4})):\rightarrow f(\frac{\pi}{4})=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

$g(\frac{\sqrt{2}}{2})=\frac{\sqrt{2}}{2}\left(\sqrt{1-(\frac{\sqrt{2}}{2})^2}\right)=\frac{\sqrt{2}}{2}\left(\sqrt{\frac{1}{2}}\right)=\frac{\sqrt{2}}{2}\times\frac{1}{\sqrt{2}}=\frac{1}{2}$

۴ (۲)

$f(x)=\{(4,5)(6,5)(8,12)(10,12)\}$

$g(x)=\{(4,6)(2,4)(6,8)(8,10)\}$

ب) $gof(x)=g(f(x)):$ $x:\ 4\xrightarrow{f(x)}5\xrightarrow{g(x)}\times$, $6\rightarrow5\rightarrow\times$, $8\rightarrow12\rightarrow\times$, $10\rightarrow12\rightarrow\times$ $\quad \varnothing$

ج) $fof(x)=f(f(x)):$ $x:\ 4\xrightarrow{f(x)}5\xrightarrow{f(x)}\times$, $6\rightarrow5\rightarrow\times$, $8\rightarrow12\rightarrow\times$, $10\rightarrow12\rightarrow\times$ $\quad \varnothing$

الف) $fog(x)=f(g(x))$ $x:\ 4\xrightarrow{g(x)}6\xrightarrow{f(x)}5$, $2\rightarrow4\rightarrow5$, $6\rightarrow8\rightarrow12$, $8\rightarrow10\rightarrow12$ $\quad\{(4,5),(2,5),(6,12),(8,12)\}$

د) $g(g(x)):$ $x:\ 4\xrightarrow{g(x)}6\xrightarrow{g(x)}8$, $2\rightarrow4\rightarrow6$, $6\rightarrow8\rightarrow10$, $8\rightarrow10\rightarrow\times$ $\quad\{(4,8),(2,6),(6,10)\}$

۵ (۲)

$f=\{(2,1)(3,2)(4,5)(1,6)\}$

$g=\{(1,2)(3,1)(a,3)(b,1)\}$

$(a,b)=\boxed{(4,5)}$

$(4,2)\in f(g(x))\Rightarrow f(g(x)):\ \begin{matrix}x\in D_g\\ g(x)\in D_f\end{matrix}\Big\}\Rightarrow\begin{matrix}4\in D_g\\ 2\in R_f\rightarrow g(x)=3\end{matrix}\quad \begin{matrix}4\in D_g\end{matrix}\Big\}(4,3)\rightarrow a=4$

$(4,1)\in g(f(x))\Rightarrow\begin{matrix}4\in D_f\\ f(x)\in D_g\end{matrix}\Big\}\Rightarrow\begin{matrix}4\in D_f\\ 1\in R_g\rightarrow f(x)=b\end{matrix}\quad\begin{matrix}4\in D_f\end{matrix}\Big\}\ f(4)=b=5\rightarrow b=5$

$f(f(x)) = 4x+3 \rightarrow f(x) = ax+b \rightarrow f(ax+b) = a(ax+b)+b = 4x+3$

$a^2x+ab+b = 4x+3$

$\begin{cases} a^2 = 4 \\ ab+b = 3 \end{cases} \rightarrow a=2 \,,\; 3b=3 \rightarrow b=1 \,,\; f(x)=2x+1 \qquad a=-2 \,,\; -b=3 \rightarrow b=-3 \,,\; f(x)=-2x-3$

$g(\underbrace{2x+3}_{t}) = 3x-2$

$2x+3=t \,,\; x=\frac{t-3}{2} \;\Big\}\; g(t) = 3\left(\frac{t-3}{2}\right)-2$

$g(x) = 3\left(\frac{x-3}{2}\right)-2 = \frac{3x-9-4}{2} = \underbrace{\frac{3x-13}{2} = g(x)}$

$\Rightarrow g(f(-1)) \rightsquigarrow f(-1) \begin{cases} 2x+1 = -2+1 = -1 \\ -2x-3 = 2-3 = -1 \end{cases} = -1 \rightarrow g(-1) = \frac{3(-1)-13}{2} = \frac{-16}{2} = -8$

۲

$f(x) = \sqrt{x+|x|} \rightarrow D_f: x+|x| \geq 0 \;\; \begin{smallmatrix} + \checkmark \\ - \checkmark \end{smallmatrix} \rightarrow D_f = \mathbb{R}$ ۲

$g(x) = \frac{1}{x^2-4x} \rightarrow D_g: x^2-4x \neq 0 \,,\; x(x-4) \neq 0 \rightarrow x\neq 0 \,,\; x \neq 4 \rightarrow D_g: \mathbb{R}-\{0,4\}$

$D_{gof}: \rightarrow \begin{cases} x \in D_f \\ f(x) \in D_g \end{cases} \Rightarrow (1)\; x \in \mathbb{R}$

$(2)\; \sqrt{x+|x|} \neq 0 \rightarrow x+|x| \neq 0 \rightarrow x = (0,+\infty)$

$\sqrt{x+|x|} \neq 4 \rightarrow x+|x| \neq 16 \rightarrow x \neq 8 \;\Big\}\; x = (0,+\infty)-\{8\}$

$(1)\cap(2) \Rightarrow \boxed{D_{gof}: (0,+\infty)-\{8\}} = \mathbb{R}^+-\{8\}$ ۷

$f(x) = \sqrt{1-x^2} \rightarrow D_f: 1-x^2 \geq 0 \,,\; x^2 \leq 1 \,,\; -1 \leq x \leq 1$

$g(x) = \sqrt{x} \rightarrow D_g: x \geq 0$

$\Big\} \Rightarrow D_{f+g} = D_f \cap D_g \Rightarrow D_{f+g}: [0,1]$ ۲

$D_{(f+g)of}: \begin{cases} x \in D_f \\ f(x) \in D_{f+g} \end{cases} \Rightarrow (1)\; -1 \leq x \leq 1 \,,\; (2)\; \underbrace{0 \leq}_{(I)} \underbrace{\sqrt{1-x^2} \leq 1}_{(II)}$

$(I)\; \sqrt{1-x^2} \geq 0 \xrightarrow{\text{توان } 2} 1-x^2 \geq 0 \rightarrow x^2 \leq 1 \,,\; -1 \leq x \leq 1$

$(II)\; \sqrt{1-x^2} \leq 1 \xrightarrow{(\,)^2} 1-x^2 \leq 1 \,,\; x^2 \geq 0 \;\; \mathbb{R} \;\Big\}\; -1 \leq x \leq 1$

$(1)\cap(2) \Rightarrow D_{(f+g)of}: \boxed{[-1,1]}$ ۸

الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5 \rightarrow \frac{3x+1}{x-2} = t \rightarrow x = -\frac{-2t-1}{t-3} = \frac{2t+1}{t-3}$

$\rightarrow f(t) = 4\left(\frac{2t+1}{t-3}\right)+5 \rightarrow f(t) = \frac{8t+4+5t-15}{t-3} \rightarrow \boxed{f(x) = \frac{13x-11}{x-3}}$

۲

ب) $f\left(\underbrace{x+\frac{1}{x}}_{t}\right) = x^3+\frac{1}{x^3} \rightarrow (x+\frac{1}{x})^3 = x^3+\frac{1}{x^3}+3x+3\frac{1}{x}$ ۹

$x^3+\frac{1}{x^3} = (x+\frac{1}{x})^3 - 3(x+\frac{1}{x}) \Rightarrow f(t) = t^3-3t$

$x^3+\frac{1}{x^3} = t^3-3t \qquad \boxed{f(x) = x^3-3x}$

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g ریشه‌های: $2\sqrt{2}\,,\,1$

۲

$f(x) = x\sqrt{x}$

اختلاف ریشه‌ها

$g(f(x))$ ؟ $\Rightarrow$ ریشه‌های $g(x) = 2\sqrt{2}\,,\,1 \rightarrow$ 10

$f(x) = 2\sqrt{2} \rightarrow x\sqrt{x} = 2\sqrt{2} \rightarrow x=2$

$f(x) = 1 \rightarrow x\sqrt{x} = 1 \rightarrow x = 1$

$\Big\}$ ریشه‌های $gof(x)$

اختلاف $= 2-1 = 1$