

$$f(x) = \begin{cases} \cot \frac{\pi x}{2} & , x \leq 1 \\ \sqrt{x^2 + 1} & , x > 1 \end{cases}$$

$$f \circ f\left(\frac{1}{r}\right) \rightarrow f\left(\frac{1}{r}\right) = \cot \frac{\pi x}{2} \rightarrow \cot \frac{\pi}{r} \rightarrow \sqrt{r}$$

$$f(\sqrt{r}) = \sqrt{(\sqrt{r})^2 + 1} = \sqrt{r+1} \quad (2)$$

$$a) f\left(\frac{1}{x}\right) = \sqrt{\frac{1-x}{x}} \quad g(x) = x \cos x$$

$$f \circ g\left(\frac{\pi}{r}\right) \rightarrow g\left(\frac{\pi}{r}\right) = x \cos x = r \times \frac{1}{2} = \frac{r}{2}$$

$$f\left(\frac{1}{x}\right) = f\left(\frac{1}{r}\right) \quad f(r) = \sqrt{\frac{r-1}{r}} = \frac{\sqrt{r}}{r}$$

$$f(x) = \sin(x) \\ g(x) = x \sqrt{1-x^2}$$

$$f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} = \frac{\sqrt{r}}{r}$$

$$g \circ f\left(\frac{\pi}{2}\right)$$

$$g\left(\frac{\sqrt{r}}{r}\right) = \frac{\sqrt{r}}{r} \sqrt{1 - \frac{r}{r^2}} = \frac{\sqrt{r}}{r} \sqrt{\frac{r-1}{r}} = \frac{1}{\sqrt{r}} \times \frac{\sqrt{r}}{r} = \frac{1}{r}$$

$$f(x) = \{(2,0)(4,0)(1,12)(1,12)\}$$

$$g(x) = \{(2,4)(2,2)(4,1)(1,1)\}$$

$$a) f \circ g(x) = \{(2,0)(2,0)(4,12)(1,12)\}$$

$$b) g \circ f(x) = \emptyset \quad c) f \circ f(x) = \emptyset$$

$$d) g \circ g(x) = \{(2,1)(2,4)(4,1)\}$$

$$f = \{(r,1)(r,r)(r,a)(1,r)\}$$

$$g = \{(1,r)(r,1)(a,r)(b,1)\}$$

$$(r,r) \in f \circ g \rightarrow g(x) = t \rightarrow f(t) = r \rightarrow t = r \rightarrow x = a \rightarrow a = r$$

$$(r,1) \in g \circ f \rightarrow g(f(x)) = 1 \rightarrow f(x) = b \rightarrow (r,0) \rightarrow b = 0 \rightarrow (r,0)$$

$$f(f(x)) = 2x + r$$

$$g(rx+r) = rx - r$$

$$g \circ f(-1) = ?$$

$$g(x) = cx + d \\ \frac{r}{r}x - \frac{1r}{r}$$

$$f(x) = 2x + 1 \rightarrow g(f(-1)) = -1$$

$$f(x) = -2x - r \rightarrow g(f(-1)) = -1$$

$$f(f(x)) = a(ax+b) + b = a^2x + ab + b = 2x + r$$

$$\begin{cases} a^2 = 2 \\ a = \pm \sqrt{2} \end{cases} \begin{cases} a = r \rightarrow b = 1 \\ a = -r \rightarrow b = -r \end{cases}$$

$$g(x) = cx + d$$

$$g(rx+r) = c(rx+r) + d = rx - r \rightarrow rc = r \rightarrow c = \frac{r}{r}$$

$$rcx + rc + d \rightarrow rc + d - r = d = -\frac{1r}{r}$$

$$f(x) = \sqrt{x+|x|}$$

$$g(x) = \frac{1}{x^2 - 2x}$$

$$f(x) = \begin{cases} \sqrt{x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

-v

$$D_{g \circ f} =$$

$$D_f = \mathbb{R}$$

$$\sqrt{x+|x|} \neq 0 \rightarrow x \geq 0$$

$$D_g = \mathbb{R} - \{0, 2\}$$

$$\sqrt{x+|x|} \neq \{0, 2\}$$

$$\sqrt{x+|x|} \neq 2 \rightarrow x \neq 1$$

$$D_{g \circ f} = (0, +\infty) - \{1\}$$

$$f(x) = \sqrt{1-x^2}$$

$$g(x) = \sqrt{x}$$

$$\{x \in D_f, f(x) \in D_{g \circ f}\}$$

-A

$$D(f \circ f + g \circ f) = ?$$

$$\sqrt{1-x^2} \in [0, 1] \Rightarrow 0 \leq \sqrt{1-x^2} \leq 1 \xrightarrow{\text{مربع}} 0 \leq 1-x^2 \leq 1$$

$$-1 \leq -x^2 \leq 0 \rightarrow 0 \leq x^2 \leq 1 \rightarrow -1 \leq x \leq 1$$

$$D_f = \frac{-1+1}{-1-1} = -1$$

$$D_f = [-1, 1]$$

$$D(f+g) = D_f \cap D_g = [-1, 1] \cap [0, +\infty)$$

$$D_g = [0, +\infty)$$

$$\rightarrow [0, 1]$$

$$\text{مثال 1) } f\left(\frac{rx+1}{x-r}\right) = x+2$$

$$\text{مثال 2) } f\left(x - \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$$

-A

$$f(x) = ? \quad \frac{rx+1}{x-r} = t$$

$$f(x) = ? \quad \left(x + \frac{1}{x}\right) = t$$

$$f(t) = x + 2 \left(\frac{t+1}{t-r}\right) + 2$$

$$t^2 = x^2 + \frac{1}{x^2} + r\left(x + \frac{1}{x}\right) \Rightarrow t^2 = x^2 + \frac{1}{x^2} + rt$$

$$x^2 + \frac{1}{x^2} = t^2 - rt$$

$$f(t) = t^2 - rt$$

$$f(x) = x^2 - rx$$

$$x = \frac{rt+1}{t-r} \rightarrow f(x) = \frac{1^2 x - 11}{x-3}$$

$$f(x) = x\sqrt{x}$$

$$x \geq 0$$

لـ إيجاد مجال دالة $f(x) = x\sqrt{x}$ نلاحظ ان $x \geq 0$ و $\sqrt{x}$ معرف لـ $x \geq 0$ فقط

$$g(f(x)) = 0$$

$$g(x) = 0$$

اقتناط لـ $g(x) = 0$ نلاحظ ان $g(x) = 0$ لـ $x=0$ و $x=4$ فقط

$$x_1 = 1 \rightarrow x_2 = 2\sqrt{2}$$

$$f(x) = \begin{cases} x_1 = 1 \rightarrow x\sqrt{x} = 1 \rightarrow x'_1 = 1 \\ x_2 = 2\sqrt{2} \rightarrow x\sqrt{x} = 4\sqrt{2} \rightarrow x'_2 = 2 \end{cases}$$

$$x'_2 - x'_1 = 2 - 1 = 1$$