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لیلا ابراهیمی تکلیف ۹

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$$f(x)=\begin{cases}\cot\frac{\pi x}{4}, & x\le 1\\ \sqrt{x^2+1}, & x>1\end{cases}$$

$f\circ f(\frac{1}{3}) \rightarrow f(\frac{1}{3})=\cot\frac{\pi x}{4} \rightarrow \cot\frac{\pi}{6} \rightarrow \sqrt{3}$

$f(\sqrt{3}) \Rightarrow \sqrt{(\sqrt{3})^2+1}$ 2

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الف) $f(\frac{1}{x})=\sqrt{\frac{2x-1}{x^2}}$ $g(x)=2\cos^2 x$

$f\circ g(\frac{\pi}{3}) \rightarrow g(\frac{\pi}{3})=2\cos^2\pi=2\times\frac{1}{4}=\frac{1}{2}$

$f(\frac{1}{x})=f(\frac{1}{2})$ $f(2)=\sqrt{\frac{4-1}{4}}=\frac{\sqrt{3}}{2}$

$x=2$

ب) $f(x)=[x]$ $g(x)=\frac{x}{1-x}$

$f\circ g(\sqrt{2}) \rightarrow g(\sqrt{2})=\frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{1+\sqrt{2}}{1+\sqrt{2}}=\frac{\sqrt{2}+2}{-1}$

$f(-\sqrt{2}-2)=[-\sqrt{2}-2]$ -4

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۳-

$f(x)=\sin(x)$

$g(x)=x\sqrt{1-x^2}$

$g\circ f(\frac{\pi}{4})$

$f(\frac{\pi}{4})=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

$g(\frac{\sqrt{2}}{2})=\frac{\sqrt{2}}{2}\sqrt{(1-\frac{2}{4})}=\frac{\sqrt{2}}{2}\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{2}=\frac{1}{2}$

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$f(x)=\{(4,5)(6,5)(8,12)(10,12)\}$

$g(x)=\{(4,6)(2,4)(6,8)(8,10)\}$

الف) $f\circ g(x)$ $\{(4,5)(2,5)(6,12)(8,12)\}$

ب) $g\circ f(x)$ $\varnothing$ ج) $f\circ f(x)$ $\varnothing$ د) $g\circ g(x)$ $\{(4,8)(2,6)(6,10)\}$

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$f=\{(2,1)(3,2)(4,a)(1,7)\}$

$g=\{(1,2)(3,1)(a,3)(b,1)\}$

$(4,2)\in f\circ g \rightarrow g(x)=t \rightarrow f(t)=2 \rightarrow t=3 \rightarrow x=a \rightarrow a=4$

$(4,1)\in g\circ f \rightarrow g(f(x))=1 \rightarrow f(x)=b \rightarrow (4,5) \rightarrow b=5 \rightarrow (4,5)$

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$f(f(x))=4x+3$

$g(2x+3)=3x-2$

$g\circ f(-1)=?$

$g(x)=cx+d$

$\frac{3}{2}x-\frac{13}{2}$

$f(x)=2x+1 \rightarrow g(f(-1)) \rightarrow -8$

$f(x)=-2x-3 \rightarrow g(f(-1)) \rightarrow -8$

$f(f(x))=a(ax+b)+b=a^2x+ab+b=4x+3$

$a^2=4$

$a=\pm 2$

$\begin{cases}a=2 \rightarrow b=1\\ a=-2 \rightarrow b=-3\end{cases}$

$g(x)=cx+d$

$g(2x+3)=c(2x+3)+d=3x-2 \Rightarrow 2c=3 \rightarrow c=\frac{3}{2}$

$2cx+3c+d \rightarrow 3c+d=-2 \quad \boxed{d=-\frac{13}{2}}$

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$f(x) = \sqrt{x+|x|}$

$g(x) = \dfrac{1}{x^2 - 4x}$

$D_{g \circ f} =$

$f(x) \begin{cases} \sqrt{2x} & x \geq 0 \\ 0 & x \leq 0 \end{cases}$

$D_f = \mathbb{R}$

$\sqrt{x+|x|} \neq 0 \rightarrow x > 0$

$\sqrt{x+|x|} \neq 4 \rightarrow x \neq 8$

$D_g = \mathbb{R} - \{0, 4\}$

$\sqrt{x+|x|} \neq \{0, 4\}$

$\boxed{D_{g \circ f} = (0, +\infty) - \{8\}}$

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$f(x) = \sqrt{1-x^2}$

$g(x) = \sqrt{x}$

$D(f \circ f + g \circ f) = ?$

$D_f = \dfrac{-1 \quad +1}{- \quad + \quad -}$ $\rightarrow$ $\boxed{D_f = [-1, 1]}$

$D_g = [0, +\infty)$

$\{x \in D_f \,,\, f(x) \in D_{f \circ g}\}$

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$\sqrt{1-x^2} \in [0, 1] \Rightarrow 0 \leq \sqrt{1-x^2} \leq 1 \xrightarrow{\text{توان ۲}} 0 \leq 1-x^2 \leq 1$

$-1 \leq -x^2 \leq 0 \rightarrow 0 \leq x^2 \leq 1 \rightarrow -1 \leq x \leq 1 \rightarrow$

$D(f+g) = D_f \cap D_g = [-1, 1] \cap [0, +\infty)$

$0 \leq \sqrt{1-x^2} \leq 1 \rightarrow x^2 \geq 0 \checkmark$

$D(f+g) \circ f = [-1, 1]$

$\rightarrow \cancel{[0, 1]}$ $\boxed{[-1, 1]}$

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الف) $f\left(\dfrac{3x+1}{x-2}\right) \Rightarrow 4x + 5$

$f(x) = ?$ $\quad \dfrac{3x+1}{x-2} = t$

$f(t) = 4\left(\dfrac{2t+1}{t-3}\right) + 5$

$x = \dfrac{2t+1}{t-3} \rightarrow f(x) = \dfrac{13x - 11}{x-3}$

ب) $f\left(x - \dfrac{1}{x}\right) = x^3 + \dfrac{1}{x^3}$

$f(x) = ?$ $\quad \left(x + \dfrac{1}{x}\right) = t$

$t^3 = x^3 + \dfrac{1}{x^3} + 3\underbrace{\left(x + \dfrac{1}{x}\right)}_{t} \Rightarrow t^3 = x^3 + \dfrac{1}{x^3} + 3t$

$x^3 + \dfrac{1}{x^3} = t^3 - 3t$

$f(t) = t^3 - 3t$

$f(x) = x^3 - 3x$

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۱۰- نمودار تابع g محور x ها را در نقاطی با طول ۱ و $2\sqrt{2}$ قطع می‌کند [illegible]

اختلاف طول نقاطی که نمودار تابع $g \circ f(x)$ محور x ها را قطع می‌کند

$f(x) = x\sqrt{x} \quad x \geq 0$

$g(f(x)) = 0 \qquad g(x) = 0$

$x_1 = 1 \,,\, x_2 = 2\sqrt{2}$

$f(x) = \begin{cases} x_1 = 1 \rightarrow x\sqrt{x} \Rightarrow 1 \rightarrow x'_1 = 1 \\ x_2 = 2\sqrt{2} \rightarrow x\sqrt{x} = 2\sqrt{2} \rightarrow x'_2 = 2 \end{cases}$

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$x'_2 - x'_1 = 2 - 1 = 1$