نام و نام خانوادگی ...حسنا رضایی....... پاسخنامه تشریحی تکلیف شماره ...9... کلاس ...یازدهم.......

۱

$$f(x) = \begin{cases} \cos\frac{\pi x}{4} & ; x \le 1 \\ \sqrt{x^2+1} & ; x > 1 \end{cases}$$

$fof\left(\frac{2}{3}\right) = ?$ $\to f\left(\frac{2}{3}\right) \to \cos\frac{\pi \times \cancel{2}}{\underbrace{3\times \cancel{4}}_{6}}$ $\cos\left(\frac{\pi}{6}\right)$ $\sqrt{3}$ $f(\sqrt{3})$

$\sqrt{x^2+1} \to \sqrt{3+1} = 2$

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الف) $f\left(\frac{1}{x}\right) = \sqrt{\frac{2x-1}{x^2}}$ و $g(x) = 2\cos^2 x \to fog\left(\frac{\pi}{4}\right) = ?$

$\frac{1}{2} = \frac{1}{x}$, $x = 2$; $\frac{2\times 2-1}{4} \to \frac{\sqrt{3}}{2}$; $\frac{2\cos^2\left(\frac{\pi}{4}\right)}{\frac{1}{2}}$

ب) $f(x) = [x]$ و $g(x) = \frac{x}{1-x} \to fog(\sqrt{2}) = ?$

$\to \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = -2-\sqrt{2} \to [-2-\sqrt{2}] \to -4$

۳

$f(x) = \sin x \to \sin\frac{\pi}{4} \to \frac{\sqrt{2}}{2}$

$g(x) = x\sqrt{1-x^2}$

$gof\left(\frac{\pi}{4}\right) = ?$

$\to \frac{\sqrt{2}}{2} \times \sqrt{1-\frac{1}{2}} = \frac{\sqrt{2}}{2} \to \frac{1}{2}$

۴

$f(x) = \{(4,5),(6,5),(8,12),(10,12)\}$ و $g(x) = \{(4,6),(2,4),(6,8),(8,10)\}$

الف) $fog(x) = \{(4,5),(2,5),(6,12),(8,12)\}$

$4\to 6\to 5$, $2\to 4\to 5$, $6\to 8\to 12$, $8\to 10\to 12$

ب) $gof(x) = \varnothing$

$4\to 5\to \times$, $6\to 5\to \times$, $8\to 12\to \times$, $10\to 12\to \times$

ج) $fof(x) = \varnothing$

$4\to 5\to \times$, $6\to 5\to \times$, $8\to 12\to \times$, $10\to 12\to \times$

د) $gog(x) = \{(4,8),(2,6),(6,10)\}$

$4\to 6\to 8$, $2\to 4\to 6$, $6\to 8\to 10$, $8\to 10\to 12$

۵

$f = \{(2,1),(3,2),(4,5),(1,7)\}$

$g = \{(1,2),(3,1),(a,3),(b,1)\}$

$(4,2) \in fog$

$(4,1) \in gof$

$(a,b) = ?$

$\to (a,b) = (4,5)$

$fog(x) \to$ $1\to 2\to 1$, $3\to 1\to 7$

$(4,2) \to \boxed{a\to 3\to 2}$ $a,2 \to (4,2)$ $\boxed{a=4}$

$b\to 1\to 7$

$gof(x)$

$(4,1) \to$ $2\to 1\to 2$, $3\to 2\to$, $\boxed{4\to 5\to}$ $\to$ $=1 \to$ وقتی به g

$1\to 7\to$

۵ را بدیم به ما ۱ بده

و طبق صورت سوال تابع g به ازای b و ۳، ما را ۱ می‌دهد پس $\boxed{5=b}$

۶

$f(f(x)) = 4x+3 \hookrightarrow a^2x+(a+1)b = 4x+3$

$\hookrightarrow a = \pm 2$

$(a+1)b = 3$

$\hookrightarrow a=2 \to b=1$

$\hookrightarrow a=-2 \to b=-3$

$\left.\begin{matrix} 2x+1 \\ -2x-3 \end{matrix}\right\} \xrightarrow{x=-1} \boxed{y=-1}$

$g(2x+3) = 3x-2$

$f \hookrightarrow x = \frac{t-3}{2} \to \frac{3t-9}{2}-2 \to \frac{3t-9-4}{2} \to \frac{3t-13}{2} \hookrightarrow \frac{3x-13}{2} = g(x)$

$g \circ f(-1) = ?$

$g(-1) \hookrightarrow \frac{3x-13}{2} \Big\}\ -8$

۷

$f(x) = \sqrt{x+|x|} \to x+|x| \geq 0 \hookrightarrow \mathbb{R}$

$g(x) = \frac{1}{x^2-4x} \to x^2-4x \neq 0 \to x(x-4) \neq 0$

$x \neq 0 , 4$

$D_{(g\circ f)} = ? \hookrightarrow x \in D_f \to \mathbb{R}$

$f(x) \in D_g \to \sqrt{x+|x|} \neq 0 , 4 \Big\} \begin{matrix} x+|x| \neq 0 \\ x+|x| \neq 16 \end{matrix} \to \begin{matrix} x \neq 0 , 8 \\ x \notin \mathbb{R}^- \end{matrix} \Big\} (0,8)\cup(8,+\infty)$

۸

$f(x) = \sqrt{1-x^2}$

$g(x) = \sqrt{x}$

$[-1,1]$

$-x^2 \leq 0$

$1-x^2 \leq 1$

$\sqrt{1-x^2} \leq 1$

$(f+g)\circ f$ دامنه ؟

$\hookrightarrow x \in D_f \to 1-x^2 \geq 0 \to 1 \geq x \geq -1$

$f(x) \in D_{(f+g)} \hookrightarrow D_{(f+g)} \to \sqrt{1-x^2}+\sqrt{x} \to 1 \geq x \geq 0 \to 0 \leq \sqrt{1-x^2} \leq 1$

۹

الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5$

$t \hookrightarrow t = \frac{3x+1}{x-2} \to -\frac{-2t-1}{t-3} \Big\}\ \frac{2t+1}{t-3} = x \xrightarrow{4x+5} \frac{2\times 4t+4}{t-3}+5 \Big\}\ \frac{8t+4+5t-15}{t-3} \to \frac{13t-11}{t-3}$

$\frac{13x-11}{x-3}$

ب) $f\left(x+\frac{1}{x}\right) = x^3+\frac{1}{x^3}$

t

x^3-3x

$\left(x+\frac{1}{x}\right)^3 = t^3 \hookrightarrow x^3+3x+\frac{3}{x}+\frac{1}{x^3} = t^3 \Big\}\ x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right) \to t^3-3t$

۱۰

$g(x) \xrightarrow{x=1} y=0$

$\xrightarrow{x=2\sqrt{2}}$

$\hookrightarrow g(f(x)) = 0 \to f(x) = 2\sqrt{2} , 1$

$\hookrightarrow x\sqrt{x} = 2\sqrt{2} \Big\}\ x_2 = 2 \to \{2 , 1\}$

$x\sqrt{x} = 1 \quad x_1 = 1$