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نام و نام خانوادگی ......بهار سعدی...... پاسخنامه تشریحی تکلیف شماره ...۸.... کلاس ...یازدهم دختر......

۱	$\cot \frac{\pi}{2} \times \frac{\pi}{3} = \cot \frac{\pi}{6} = \sqrt{3} \xrightarrow{>} \sqrt{(\sqrt{3})^2+1} = 2$ 2, <1 ۲
۲	الف) $2\times\left(\frac{1}{2}\right)^2 = \frac{1}{2} = g\left(\frac{\pi}{3}\right) \Rightarrow \frac{1}{x} \xrightarrow{x=\frac{1}{2}} 2 \to f(2) = \frac{\sqrt{3}}{2}$ ۲ ب) $g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = -\sqrt{2}-2$ $[-\sqrt{2}-2] = -4$
۳	$f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \quad g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4}$ $= \frac{1}{2}$ ۲
۴	الف) $fog(x) = 4 \to 6 \to 5 \,/\, 2 \to 4 \to 5 \,/\, 6 \to 8 \to 12 \,/\, 8 \to 10 \to 14$ $= \{(4,5),(2,5),(6,12),(8,12)\}$ ۲ ب) $gof(x) = 4 \to 5 \to x \,/\, 6 \to 5 \to x \,/\, 8 \to 12 \to x \,/\, 10 \to 12 \to x = \{\emptyset\}$ ج) $fof(x) = 4 \to 5 \to x \,/\, 6 \to 5 \to x \,/\, 8 \to 12 \to x \,/\, 10 \to 12 \to x = \{\emptyset\}$ د) $gog(x) = 4 \to 6 \to 8 \,/\, 2 \to 4 \to 6 \,/\, 6 \to 8 \to 10 \,/\, 8 \to 10 \to x$ پس $= \{(4,8),(2,6),(6,10)\}$
۵	$f(\underbrace{g(4)}_{3}) = 2 \to g(4) = 3 \to \underline{a = 4}$ $g(\underbrace{f(14)}_{4}) = 1 \to f(4) = 5 \to \underline{b = 5}$ ۲ $(a,b) = (4,5)$

$a(ax+b)+b = a^2x + ab + b = 4x+3$ $\quad f(-1)=-1$

$a^2 = 4 \to a=2 \to 2b+b=3 \to b=1 \xrightarrow{①} f(x)=2x+1$ ✓

$\searrow a=-2 \to -2b+b=3 \to b=-3 \xrightarrow{②} f(x)=-2x+3 \to f(-1)=-1$

$2x+3=t \to x=\frac{t-3}{2} \to g(t)=3\left(\frac{t-3}{2}\right)-2 = \frac{3t-13}{2}$

$g\circ f(-1) = g(\underbrace{f^{-1}(-1)}) \to g(-1) = \frac{3(-1)-13}{2} = \underbrace{-8}$

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$\left.\begin{array}{l} x\in D_f \\ f(x)\in D_g \end{array}\right\} \to$ $D_f \Rightarrow x+|x|\geq 0 \to x \geq -|x| \to D_f = \mathbb{R}$

$\to x^2-4x \neq 0 \to D_g = \mathbb{R}-\{4,0\}$

$\left.\begin{array}{l} x+|x|\neq 0 \to x \neq -|x| \to x>0 \\ x+|x| \neq 16 \to x\neq 8 \end{array}\right\} D_{g\circ f} = (0,+\infty)-\{8\}$

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$\sqrt{1-x^2} - \sqrt{x} \xrightarrow{\sqrt{1-x^2}} \sqrt{1-1+x^2} - \sqrt{1-x^2} = \sqrt{x^2} + \sqrt{1-x^2}$

$= x + \sqrt{1-x^2} \to 1-x^2 \geq 0 \to x^2 \leq 1 \to -1 \leq x \leq 1$

$\Rightarrow D_f = [-1,1]$

$D_f = -1\leq x\leq 1 \cap D_g = x\geq 0 \to [0,1]$ $\quad 0\leq\sqrt{1-x^2}\leq 1 \to x\geq 0$ ✓

$D_{(f+g)of} = [-1,1]$

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الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5 \to \frac{3x+1}{x-2} = t \to tx-2t = 3x+1 \to 3x-tx-2t-1 \to x(3-t) = -t-1$ $\to x = \frac{2t+1}{t-3}$

$f(t) = 4\times\frac{2t+1}{t-3} + 5 = \frac{13t-11}{t-3} \to f(x) = \frac{13x-11}{x-3}$

ب) $f(\underbrace{x+\frac{1}{x}}_{t}) = x^3 + \frac{1}{x^3} \to \left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x\times\frac{1}{x}\left(x+\frac{1}{x}\right)$

$\to x^3+\frac{1}{x^3} = t^3-3t$

$\to f(t) = t^3-3t \to f(x) = x^3-3x$

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$g(1)=0 \to g(2\sqrt{2})=0 \to f(x) = x\sqrt{x}$

$g\circ f(x) = g(f(x)) = 0 \to f(x)=1 \to x=1$

$\searrow f(x) = 2\sqrt{2} \to x=2$ $\Big\}$ $2-1=1$

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