یازدهم تجربی

بنام خداوند دانش

Subject: (ریاضی مثلثات ۹)

Date: برنیا خانجستانی

①

$$f(x)=\begin{cases}\cot\frac{\pi x}{4} & : x\le 1\\ \sqrt{x^2+1} & : x>1\end{cases}$$

$f(f(\frac{2}{3}))$ $\qquad \frac{2}{3}<1 \Rightarrow f(\frac{2}{3})=\cot\frac{\pi\times 2}{4\times 3}=\cot\frac{\pi}{6}$

$=\sqrt{3}$

$\Rightarrow \sqrt{3}>1 \quad f(\sqrt{3})=\sqrt{\underbrace{(\sqrt{3})^2+1}_{4}}=2$

②

الف) $f(\frac{1}{x})=\sqrt{\frac{2x-1}{x^2}}$, $g(x)=2\cos^2 x$

$f(g(x))$ $\qquad f(g(\frac{\pi}{3}))$ $\qquad g(\frac{\pi}{3})=2\underbrace{\cos^2\frac{\pi}{3}}_{\frac{1}{4}}$ $=\frac{1}{2}$

$f(\frac{1}{2}) \Rightarrow x=2 \Rightarrow \sqrt{\frac{2\times 2-1}{4}}=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$

ب) اگر $f(x)=[x]$ و $g(x)=\frac{x}{1-x}$

$f(g(\sqrt{2}))=$

$g(\sqrt{2})=\frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{1+\sqrt{2}}{1+\sqrt{2}}=\frac{\sqrt{2}+2}{-1}=-\sqrt{2}-2$

$f(\underset{\underset{-3.4}{-1.4-2}}{-\sqrt{2}-2})=[-3.4]=-4$

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بنام خداوند دانش

Subject : ( )　　　　　　Date :

③ $f(x)=\sin x$　　$g(x)=x\sqrt{1-x^2}$

$g(f(\frac{\pi}{4}))=$

$f(\frac{\pi}{4})=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2} \Rightarrow g(\frac{\sqrt{2}}{2})=\frac{\sqrt{2}}{2}\sqrt{1-\underbrace{(\frac{\sqrt{2}}{2})^2}_{\frac{2}{4}}}$

$=\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{2})=\frac{1}{2}$

④

$f(x)=\{(4,5),(6,5),(8,12),(10,12)\}$

$g(x)=\{(4,6),(2,4),(6,8),(8,10)\}$

الف) $fog(x)=\{(4,5),(2,5),(6,12),(8,12)\}$

ب) $gof(x)=\varnothing$

ج) $fof(x)=\varnothing$

د) $gog(x)=\{(4,8),(2,6),(6,10),\cancel{8}\}$

⑤ $(\overset{4}{a},\overset{5}{b})$

$f=\{(2,1),(3,2),(4,5),(1,7)\}$　　$(4,2)\in fog$

$g=\{(1,2)(3,1)(\underset{4}{a},3)(b,1)\}$　　$(4,1)\in gof$

$f(g(\overset{4}{a}))=2 \Rightarrow g(4)=3 \Rightarrow a=4$

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$g(\underbrace{f(4)}_{5})=1 \Rightarrow g(5)=1 \Rightarrow b=5$

بنام خداوند دانش

Subject : (   )   Date : ....

(۶)

f,g خطی

$f(f(x)) = 4x+3$

$g(2x+3) = 3x-2$

$f(x) = ax+b$

$g(x) = cx+d$

$g\circ f(-1)$

$\Rightarrow \boxed{g(x) = \frac{3}{2}x - \frac{13}{2}}$

$f(f(x)) = a(ax+b)+b$

$= \underbrace{a^2}_{4}x + \underbrace{ab+b}_{}$

$ab+b = 3$

$b(a+1) = 3$

$a^2 = 4 \Rightarrow a = \pm 2$

$\Rightarrow a = 2 \quad b(3) = 3 \Rightarrow \boxed{b = 1} \quad \boxed{2x+1}$

$\Rightarrow a = -2 \quad b(-1) = 3 \Rightarrow \boxed{b = -3} \quad \boxed{-2x-3}$

$g(2x+3) = c(2x+3)+d = \underbrace{2cx}_{3x} + \underbrace{3c+d}_{-2}$

$2c = 3 \Rightarrow \boxed{c = \frac{3}{2}}$

$3c+d = -2 \Rightarrow 3\left(\frac{3}{2}\right)+d = -2$

$\frac{9}{2} + d = -2$

$d = \frac{-4}{2} - \frac{9}{2} = -\frac{13}{2}$

$g(\overbrace{f(-1)}^{-1}) \Rightarrow g(-1) = -\frac{3}{2} - \frac{13}{2} = -\frac{16}{2}$

$= -8$

$f(-1) = \underbrace{2(-1)+1}_{-1}$ یا $\underbrace{-2(-1)-3}_{-1}$

به نام خداوند دانش

Subject: ( ) Date:

۷

$g(f(x)) \Rightarrow D_g \Rightarrow x^2-4x \neq 0 \Rightarrow x(x-4) \neq 0$

$\boxed{x \neq 0, 4}$

$f(x) \neq 0, 4 \qquad \sqrt{x+|x|} \neq 0, 4$

$x+|x|$ $\quad$ 0 $\quad$ $x-x=0$ $\quad$ $2x$

$x+|x| \neq 0 \Rightarrow$

$\boxed{x \neq (-\infty, 0]}$

$x+|x| \neq 16 \Rightarrow 2x \neq 16$

$\Rightarrow \boxed{x \neq 8}$

$D_{gof} = (0, +\infty) - \{8\}$

۸

$f(x) = \sqrt{1-x^2} \qquad g(x) = \sqrt{x}$

$(f+g)of = f+g(f(x)) \qquad f+g = \sqrt{1-x^2} + \sqrt{x}$

$D_{f+g} = 0 \leq x \leq 1$

$1-x^2 \geq 0 \Rightarrow (1-x)(1+x) \geq 0$

$-1 \quad 1$

$- \quad + \quad -$

$x \geq 0$

$-1 \leq x \leq 1$ $\Big\} \cap \quad 0 \leq x \leq 1$

$\Rightarrow 0 \leq f(x) \leq 1 \Rightarrow 0 \leq \sqrt{1-x^2} \leq 1 \Rightarrow 0 \leq 1-x^2 \leq 1$

$-1 \leq -x^2 \leq 0$

$\Rightarrow 1 \geq x^2 \geq 0 \Rightarrow \boxed{\begin{matrix} 0 \leq x \leq 1 \\ -1 \leq x \leq 0 \end{matrix}}$

$D_{(f+g)of} = [-1, 1]$

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به نام خداوند دانش

Subject : (   )   Date : ....

(9)

الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5$

$$t = \frac{3x+1}{x-2} \Rightarrow x = \frac{\overbrace{-(-2t-1)}^{2t+1}}{t-3}$$

$$f(t) = 4x+5 = 4\left(\frac{2t+1}{t-3}\right) + 5 = \frac{8t+4}{t-3} + \frac{5t-15}{t-3}$$

$$= \frac{13t-11}{t-3} \Rightarrow f(x) = \frac{13x-11}{x-3}$$

ب) $f\left(x+\frac{1}{x}\right) = x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3x\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$

$x+\frac{1}{x} = t$

$$= t^3 - 3t$$

$$f(t) = t^3-3t \Rightarrow f(x) = x^3-3x$$

(10)

$$g(f(x)) = 0 \Rightarrow f(x) = 1 \text{ یا } 2\sqrt{2}$$

$x\sqrt{x} = 1 \Rightarrow x = 1$ $2-1=1$

$x\sqrt{x} = 2\sqrt{2} \Rightarrow x = 2$