۱

$f(\frac{\pi}{4}) = \cot \frac{\pi}{12}\times\frac{2}{4} = \cot\frac{\pi}{6} = \sqrt{3}$

$f(\sqrt{3}) = \sqrt{(\sqrt{3})^2+1} = 2$]

۲

الف) $g(\frac{\pi}{3}) = 2\times\frac{1}{2\cdot 2} \Rightarrow g(\frac{\pi}{3}) = \frac{1}{2} \rightarrow f(\frac{1}{2}) = \sqrt{\frac{4-1}{4}} = \frac{\sqrt{3}}{2}$]

دامنه $\frac{1}{x} = \frac{1}{2} \Rightarrow x=2$ پس

ب) $g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{(1+\sqrt{2})}{(1+\sqrt{2})} = \frac{\sqrt{2}+2}{\underbrace{1-2}_{-1}} = -2-\sqrt{2} \rightarrow fog(\sqrt{2}) = [\underbrace{-2-\sqrt{2}}_{-3.4}] = -4$]

۳

$f(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \Rightarrow g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\times\frac{\sqrt{\frac{1}{2}}}{1} = \frac{\sqrt{2\times\frac{1}{2}}}{2} = \frac{\sqrt{1}}{2} = \frac{1}{2}$]

۴

الف)
$2 \xrightarrow{g(x)} 6 \xrightarrow{f(x)} 5$
$3 \rightarrow 4 \rightarrow 5$
$6 \rightarrow 8 \rightarrow 12$
$8 \rightarrow 10 \rightarrow 12$
$\Rightarrow \{(2,5),(3,5),(6,12),(8,12)\}$

ب)
$4 \xrightarrow{f(x)} 5 \xrightarrow{g(x)} \emptyset$
$6 \rightarrow 5 \rightarrow \emptyset$
$8 \rightarrow 12 \rightarrow \emptyset$
$10 \rightarrow 12 \rightarrow \emptyset$
$\Rightarrow \emptyset$

ج)
$4 \rightarrow 5 \rightarrow \emptyset$
$6 \rightarrow 5 \rightarrow \emptyset$
$8 \rightarrow 12 \rightarrow \emptyset$
$10 \rightarrow 12 \rightarrow \emptyset$
$\Rightarrow \emptyset$

د)
$2 \rightarrow 6 \rightarrow 8$
$3 \rightarrow 4 \rightarrow 6$
$6 \rightarrow 8 \rightarrow 10$
$8 \rightarrow 10 \rightarrow \emptyset$
$\Rightarrow \{(4,8),(3,6),(6,10)\}$

۵

$(4,2)\in fog \Rightarrow g(4) = m,\ f(m) = 2 \xrightarrow{f(x) \text{ در}} m = 3 \xrightarrow{g(x) \text{ در}} (a,3) = (4,3) \Rightarrow a = 4$]

$(4,1)\in gof \Rightarrow f(4) = n,\ g(n) = 1 \Rightarrow n = 5 \Rightarrow (b,1) = (5,1) \Rightarrow b = 5$]

$\Rightarrow (a,b) = (4,5)$

۶

$f(x) = ax+b \Rightarrow f(f(x)) = a(ax+b)+b = a^2x+ab+b = 4x+3 \Rightarrow \begin{cases} a=2,\ b=1 & ① \\ a=-2,\ b=-3 & ② \end{cases}$

if ① $\Rightarrow f(x) = 2x+1 \rightarrow f(-1) = -1$

if ② $\Rightarrow f(x) = -2x-3 \rightarrow f(-1) = -1$

$\star\ g(-1) = -\frac{3}{2}-\frac{13}{2} = -8$]

x در هر دو حالت ثابت

$g(2x+3) = 3x-2 \rightarrow g(t) = ct+d$

$3x-2 = 3c+2cx+d \Rightarrow c = \frac{3}{2},\ d = \frac{13}{2} \Rightarrow g(t) = \frac{3}{2}t-\frac{13}{2}\ \star$

$g\circ f(x): D=\{x\in D_f,\ f(x)\in D_g\}$

✓ $D_f: x\geq -|x| \Rightarrow \mathbb{R}$ $\quad f(x)\neq ۰ \Rightarrow x<۰$

$D_g: \mathbb{R}-\{۰,۴\}$ $\quad f(x)\neq ۴ \Rightarrow x\neq ۸$ $\Big\} \Rightarrow D_{g\circ f}=(۰,+\infty)-\{۸\}$

۸

$f+g=\sqrt{1-x^۲}+\sqrt{x} \to D_{f+g}=[-1,+\infty)\cap[۰,+\infty)=\mathbb{R}^+=[۰,+\infty)$

$D=\{x\in D_f,\ f(x)\in D_{f+g}\} \Rightarrow [-1,+1]$

$[-1,1] \cap [۰,+\infty)$

۹

الف) $\frac{۳x+1}{x-۲}=t \Rightarrow \frac{۲t+1}{t-۳}$ $\quad f(t)=۴\left(\frac{۲t+1}{t-۳}\right)+۵=\frac{۸t+۴+۵t-۱۵}{t-۳}=\frac{۱۳t-۱۱}{t-۳}$

$\Rightarrow f(x)=\frac{۱۳x-۱۱}{x-۳}$

ب) $x+\frac{1}{x}=t \to f\left(x+\frac{1}{x}\right)=\left(x+\frac{1}{x}\right)^۳-۳\left(x+\frac{1}{x}\right)=t^۳-۳t$

$\Rightarrow f(x)=x^۳-۳x$

۱۰

$|^{۱}_{۰} \quad , \quad |^{۲\sqrt{۲}}_{۰}$ $\qquad g\circ f(x): g(\square)=۰ \Rightarrow \square=۲\sqrt{۲},\ \square=۱$ ★

صفر شد

$*: ۲\sqrt{۲}=x\sqrt{x} \Rightarrow x=۲$

$x\sqrt{x}=۱ \Rightarrow x=۱$ $\Big\} \to$ اختلاف طول‌ها $=۲-۱=۱$