

$$f\left(\frac{\pi}{4}\right) = \cot \frac{\pi}{4} = \cot \frac{\pi}{4} = \sqrt{1} = 1$$

$$f(\sqrt{2}) = \sqrt{(\sqrt{2})^2 + 1} = \sqrt{3}$$

$$g\left(\frac{\pi}{4}\right) = 2 \times \frac{1}{2} = 1 \Rightarrow g\left(\frac{\pi}{4}\right) = 1 \rightarrow f\left(\frac{1}{2}\right) = \sqrt{\frac{1}{4} + 1} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$$

$$\Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{(1+\sqrt{2})}{(1+\sqrt{2})} = \frac{\sqrt{2}+2}{1-2} = -2-\sqrt{2} \rightarrow f_{\circ}g(\sqrt{2}) = [-2-\sqrt{2}] = -2-\sqrt{2}$$

$$f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} = 1 \Rightarrow g\left(\frac{\pi}{2}\right) = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{1} = \frac{2}{2} = 1$$

$$\begin{array}{l} \text{الف) } f: \mathbb{R} \rightarrow \mathbb{R} \\ 2 \rightarrow 4 \rightarrow 2 \\ 4 \rightarrow 8 \rightarrow 12 \\ 8 \rightarrow 16 \rightarrow 12 \end{array} \Rightarrow \left\{ (2, 2), (4, 2), (8, 12), (16, 12) \right\}$$

$$\Rightarrow f: \mathbb{R} \rightarrow \mathbb{R} \\ 2 \rightarrow 4 \rightarrow \emptyset \\ 4 \rightarrow 8 \rightarrow \emptyset \\ 8 \rightarrow 12 \rightarrow \emptyset \\ 16 \rightarrow 12 \rightarrow \emptyset \Rightarrow \emptyset$$

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$$\Rightarrow \left\{ \begin{array}{l} (2, 4) \\ (4, 8) \\ (8, 12) \end{array} \right\}$$

$$\begin{aligned} (2, 2) \in f \circ g &\Rightarrow g(2) = m, f(m) = 2 \Rightarrow (a, 2) = (2, 2) \\ (2, 1) \in g \circ f &\Rightarrow f(2) = n, g(n) = 1 \Rightarrow (b, 1) = (2, 1) \Rightarrow b = 2 \\ &\Rightarrow (a, b) = (2, 2) \end{aligned}$$

$$f(x) = ax + b \Rightarrow f(f(x)) = a(ax + b) + b = a^2x + ab + b = ax + b \Rightarrow \begin{cases} a^2 = a \\ a = -1, b = -1 \end{cases}$$

$$\text{if } \textcircled{1} \Rightarrow f(x) = x + 1 \rightarrow f(-1) = 0$$

$$\text{if } \textcircled{2} \Rightarrow f(x) = x - 1 \rightarrow f(-1) = -2$$

$$g(x+1) = x-1 \Rightarrow g(t) = t-2$$

$$x-1 = t-2 \Rightarrow t = x+1 \Rightarrow g(t) = \frac{1}{2}t - \frac{3}{2}$$

$$* g(-1) = -\frac{1}{2} - \frac{3}{2} = -2$$

$$g \circ f(x) : D, \{x \in D_f, f(x) \in D_g\}$$

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✓ $D_f: x \geq -|x| \Rightarrow \mathbb{R}$ $f(x) \neq 0 \Rightarrow x < 0$
 $D_g: \mathbb{R} - \{0, 1\}$ $g(x) \neq 2 \Rightarrow x \neq 1$ } $\Rightarrow D_{g \circ f} = (0, +\infty) - \{1\}$

^ $f+g = \sqrt{1-x^2} + \sqrt{x} \rightarrow D_{f+g}: [-1, +\infty) \cap [0, +\infty) \cap \mathbb{R}^+ = [0, +\infty)$

$D = \{x \in D_f, f(x) \in D_{f+g}\} \Rightarrow [-1, +1] \cap [0, +\infty)$

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ا) $\frac{3x+1}{x-2} \leq t \Rightarrow \frac{3x+1}{t-2}$ $f(t) = f\left(\frac{3t+1}{t-2}\right) + x = \frac{1t+2+14t-12}{t-2} = \frac{13t-11}{t-2}$

$\Rightarrow f(x) = \frac{13x+11}{x-2}$

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ب) $x + \frac{1}{x} \leq t \rightarrow f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) \leq t^3 - 3t$

$\Rightarrow f(x) \leq x^3 - 3x$

$\left| \cdot \right|, \left| \cdot \right|^{2\sqrt{2}}$

$g \circ f(x) : g(\square) \leq 0 \Rightarrow \square \leq 2\sqrt{2}, \square \geq 1$ *

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* : $2\sqrt{2} \leq x\sqrt{x} \Rightarrow x \geq 2$ $x\sqrt{x} \leq 1 \Rightarrow x \leq 1$ } $\rightarrow$ $\{x \mid 2-1 \leq x \leq 1\}$