الینا جلالی - کلید شماره ۹

۱-

$f(\frac{\pi}{3}) = \cot\frac{\frac{\pi}{3}}{2} \times \frac{3}{4} = \cot\frac{\pi}{6} = \sqrt{3}$

$f(\sqrt{3}) = \sqrt{(\sqrt{3})^2 + 1} = \sqrt{4} = 2$ $\quad f \circ f(\frac{\pi}{3}) = 2$

۲- الف)

$f(\frac{1}{x}) = \sqrt{\frac{2x-1}{x^2}} \xrightarrow{\frac{1}{x}} f(x) = \sqrt{\frac{\frac{2}{x}-1}{\frac{1}{x^2}}} = \sqrt{\frac{\frac{2-x}{x}}{\frac{1}{x^2}}} = \sqrt{x(2-x)}$

$g(\frac{\pi}{3}) = 2\cos^2\frac{\pi}{3} = \frac{1}{2}$

$f(\frac{1}{2}) = \sqrt{\frac{1}{2}(2-\frac{1}{2})} = \sqrt{\frac{1}{2}\times\frac{3}{2}} = \frac{\sqrt{3}}{2}$ $\quad f \circ g(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

ب)

$g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = -\sqrt{2}-2$

$f(-\sqrt{2}-2) = [\underbrace{-\sqrt{2}-2}_{-3/..}] = -4$ $\quad f \circ g(\sqrt{2}) = -4$

۳-

$f(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$

۴-

الف) $f \circ g(x) = \{(1,5)(2,5)(7,12)(8,12)\}$

ب) $g \circ f(x) = \emptyset$ $\quad f(x) \notin D_g$

ج) $f \circ f(x) = \emptyset$ $\quad f(x) \notin D_f$

د) $g \circ g(x) = \{(1,8)(3,7)(4,10)\}$

۵-

$f(2) = 3 \Rightarrow g(x) = 3 \quad g(a) = 3 \quad a = 4$ $\quad$ ورودی = ۲

$g(3) = g(b) = 1 \quad f(x) = 3$ ✗

$f(x) = b$ $\quad$ ورودی = ۱ $\quad f(1) = 5 = b$

$(a, b) = (4, 5)$

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$f(x)=ax+b \quad a(ax+b)+b=4x+3$; $a^2x+ab+b=4x+3 \quad a=\pm 2$; $2 \Rightarrow b=1 \to 2x+1$; $-2 \Rightarrow b=-3 \to -2x-3$

$2x+3=t \quad x=\frac{t-3}{2}$

$g(t)=3\left(\frac{t-3}{2}\right)-2=\frac{3t-9-4}{2}=\frac{3t-13}{2}$

$f(-1)=-1 \quad g(-1)=\frac{-3-13}{2}=-\frac{16}{2}=\boxed{-8}$

با سردو نقاط

-۷

$x\in D_f \quad , \quad f(x)\in D_g$

$\mathbb{R}$

$\sqrt{x+|x|}\neq 0,2$ $\quad x+|x|\neq 0 \to \mathbb{R}^+$

$x+|x|\neq 16$

$|x|\neq 16-x$

$x\neq 16-x \quad \boxed{x\neq 8}$

$x\neq -16+x$ ×

$D_{g\circ f}=(0,+\infty)-\{8\}$

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$x\in D_f \quad f(x)\in D_{g\circ f} \quad D_{g\circ f}=[0,1]$

$[-1,1]$

$0\leq\sqrt{1-x^2}\leq 1 \quad 1-x^2\leq 1$

همواره برقرار $\quad x^2\geq 0$ همواره برقرار

$D_{(f+g)\circ f}=[-1,1]$

-۹

الف) $f\left(\frac{3x+1}{x-2}\right)=4x+5 \quad x=\frac{-2t-1}{t-3}=\frac{2t+1}{t-3} \quad f(x)=\frac{8x+4}{x-3}+5$

ب) $f\left(x+\frac{1}{x}\right)=x^3+\frac{1}{x^3} \quad t^3=\frac{1}{x^3}+x^3+3\left(x+\frac{1}{x}\right) \quad f(x)=x^3-3x$

-۱۰

$f(x)=1 \quad \sqrt{x} \quad x^2+x=1 \quad \boxed{x=1} \quad 2-1=1$

$f(x)=2\sqrt{2} \quad \sqrt{x} \quad \boxed{x=2}$