۱۹,۷۵

سارینا اسپنانی - یازدهم تجربی C - شماره تکلیف ۹

①

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & x \leqslant 1 \\ \sqrt{x^2+1} & x > 1 \end{cases} \qquad f\circ f\left(\frac{2}{3}\right) = 2$$

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$$\cot \frac{\frac{2\pi}{3}}{\frac{4}{1}} = \frac{\pi}{6} \rightarrow \sqrt{3}$$

$$\sqrt{(\sqrt{3})^2+1} = \sqrt{4} = 2$$

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②

الف) $g(x) = 2\cos^2 x$, $f\left(\frac{1}{x}\right) = \sqrt{\frac{2x-1}{x^2}}$ $\qquad f\circ g\left(\frac{\pi}{3}\right) = ?$

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$$g\left(\frac{\pi}{3}\right) = 2\left(\frac{1}{2}\right)^2 = 2 \times \frac{1}{4} = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = \sqrt{\frac{4-1}{4}} = \sqrt{\frac{3}{4}} \qquad f\circ g\left(\frac{\pi}{3}\right) = \sqrt{\frac{3}{4}}$$

ب) $f(x) = [x]$, $g(x) = \frac{x}{1-x}$ $\qquad f\circ g(\sqrt{2}) = ?$

$$g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{(1+\sqrt{2})}{(1+\sqrt{2})} = \frac{\sqrt{2}+2}{1-2} = -\sqrt{2}-2$$

$$f(-\sqrt{2}-2) = [-\sqrt{2}-2] = \underbrace{[-\sqrt{2}]}_{-2}-2 = -4 \qquad f\circ g(\sqrt{2}) = -4$$

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③

$f(x) = \sin x \qquad g(x) = x\sqrt{1-x^2}$

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$$g\circ f\left(\frac{\pi}{4}\right) = \frac{1}{2}$$

$$f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \times \sqrt{1-\frac{2}{4}} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$$

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$g(x) = \{(۴,۶),(۲,۴),(۶,۸),(۸,۱۰)\}$

$f(x) = \{(۴,۵),(۶,۵),(۸,۱۲),(۱۰,۱۲)\}$

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الف) $fog(x) \to \{(۴,۵),(۲,۵),(۶,۱۲),(۸,۱۲)\}$

ب) $gof(x) \to \varnothing$

ج) $fof(x) \to \varnothing$

د) $gog(x) \to \{(۴,۸),(۲,۶),(۶,۱۰)\}$

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$f = \{(۲,۱),(۳,۲),(۴,۵),(۱,۷)\}$

$g = \{(۱,۲),(۳,۱),(a,۳),(b,۱)\}$

$(۴,۲) \in fog \qquad \boxed{۴ = a} \to ۳ \to ۲$

$(۴,۱) \in gof \qquad ۴ \to ۵ \to ۱$

$\boxed{b = ۵}$

$(a,b) \to (۴,۵)$

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۶

$g(۲x+۳) = ۳x-۲ \qquad f(f(x)) = ۴x+۳ \qquad ۲x+۳=-۱ \quad ۲x=-۴ \to \boxed{x=-۲}$

$gof(-۱) = ? \Rightarrow$ $f(-۱) = ۲(-۱)+۱ = -۱$; $f(-۱) = -۲(-۱)-۳ = -۱$ $\to g(-۱) = ۳(-۲)-۲ = -۸$

$f(x) = ax+b \to fof = a(ax+b)+b = a^۲x+ab+b = ۴x+۳$

$a^۲ = ۴ \to \boxed{a=\pm۲} \qquad ab+b=۳$: $a=۲ \Rightarrow ۳b=۳ \to \boxed{b=۱}$; $a=-۲ \Rightarrow -b=۳ \to \boxed{b=-۳}$

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$f(x)=\sqrt{x+|x|} \qquad g(x)=\dfrac{1}{x^{2}-4x}$ ۷

$D_f \to x+|x|\geq 0$

$x+|x|\geq 0 \to D_f = \mathbb{R}$ $\qquad \sqrt{x+|x|}\neq 4$

$D_g \Rightarrow x^{2}-4x\neq 0 \quad x(x-4)\neq 0 \quad 0 \quad 4$ $\qquad x+|x|\neq 16$

$D_g=\mathbb{R}-\{0,4\}$ $\qquad \to x\neq 8$

$\therefore f(x)\in D_g$ $\qquad D_{g\circ f}=(0,+\infty)-\{8\}$

$f(x)\neq 0,4 \quad \to x\neq \mathbb{R}^{-},0,8$ $\qquad \{x\in D_f \mid f(x)\in D_g\}$

$f(x)=\sqrt{1-x^{2}} \qquad g(x)=\sqrt{x}$ $\qquad D_f \to 1-x^{2}\geq 0$ ۸

$D_{(f+g)\circ f}=[-1,1] \Leftarrow \varnothing\; R_f\in[-1,1]$ $\qquad 1\geq x^{2}$

$R_f=[0,1]$

$-1\leq x\leq 1 \to D_f=[-1,1]$

$R_f \Rightarrow y=1-x^{2} \Rightarrow x^{2}=1-y \to 1-y\geq 0 \to y\geq 1$ $\qquad D_g = x\geq 0 \to [0,+\infty)$

$D_{f+g}=D_f\cap D_g = [-1,1]\cap[0,+\infty) = [-1,1] \to R_f\subseteq[-1,1]$

$D_{(f+g)\circ f}=\{x\in D_f \mid f(x)\in D_{f+g}\}$

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الف) $f\left(\dfrac{3x+1}{x-2}\right)=4x+5$

$\dfrac{3x+1}{x-2}=t \to x=\dfrac{-(-2t-1)}{+t-3}=\dfrac{+2t+1}{t-3}$

$f(t)=4\left(\dfrac{2t+1}{t-3}\right)+5=\dfrac{4+8t+5t-15}{t-3}=\dfrac{13t-11}{t-3} \to f(x)=\dfrac{13x-11}{x-3}$

ب) $f\left(x+\dfrac{1}{x}\right)=x^{3}+\dfrac{1}{x^{3}}$

$f(t)=t^{3}-3t \to f(x)=x^{3}-3x$

$x+\dfrac{1}{x}=t \to \left(x+\dfrac{1}{x}\right)^{3}=x^{3}+\dfrac{1}{x^{3}}+3x+\dfrac{3}{x}$

$\to \left(x+\dfrac{1}{x}\right)^{3}-3\left(x+\dfrac{1}{x}\right)$

$f(x) = x\sqrt{x}$ $\qquad\qquad$ $g(f(x))$ $\qquad\qquad$ 10

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if $x = 1$, $۲\sqrt{۲}$ $\Rightarrow g(x) = 0$

$f(x) = 1 \longrightarrow x\sqrt{x} = 1 \longrightarrow \boxed{x = 1}$

$f(x) = ۲\sqrt{۲} \longrightarrow x\sqrt{x} = ۲\sqrt{۲} \longrightarrow \boxed{x = ۲}$ $\qquad$ $۲ - ۱ = ۱$