(۱)

$f(x) = \sqrt{1-x^2} = \sqrt{(1-x)(1+x)}$ $\quad \begin{array}{c} -1 \quad 1 \\ - \phi + \phi - \end{array} \quad D_f = [-1,1]$

(۲)

$3f(x) = \{(-1,0),(0,3),(1,0)\}$ $\quad 2g = \{(-1,2),(0,8),(2,0),(1,4)\}$

$2g - 3f = \{(-1,2),(0,5),(1,4)\}$ $\quad 2+5+4 = 11$

---

(۲)

(۲)

$f(x) = 2x-1 \Rightarrow f(3) = 3\times 2 - 1 = 5 \rightarrow R_f = [5,+\infty)$

$g(x) = \frac{1}{3}x + 3 \Rightarrow f(3) = \frac{1}{3}\times 3 + 3 = 4 \rightarrow R_g = (-\infty, 4]$

$R_f \cup R_g = (-\infty,4] \cup [5,+\infty)$

---

(۳)

(۲)

$\frac{-x^2}{2} + x + 3 > \frac{3}{2} \rightarrow -x^2 + 2x + 6 > 3 \rightarrow -(x^2-2x-3) > 0$

$\rightarrow x^2 - 2x - 3 > 0 \rightarrow (x+1)(x-3) > 0$ $\quad \begin{array}{c} -1 \quad 3 \\ - \phi + \phi - \end{array}$

$(a,b) = (-1,3)$

$\max \sqrt{3-(-1)} = \sqrt{4} = 2$

---

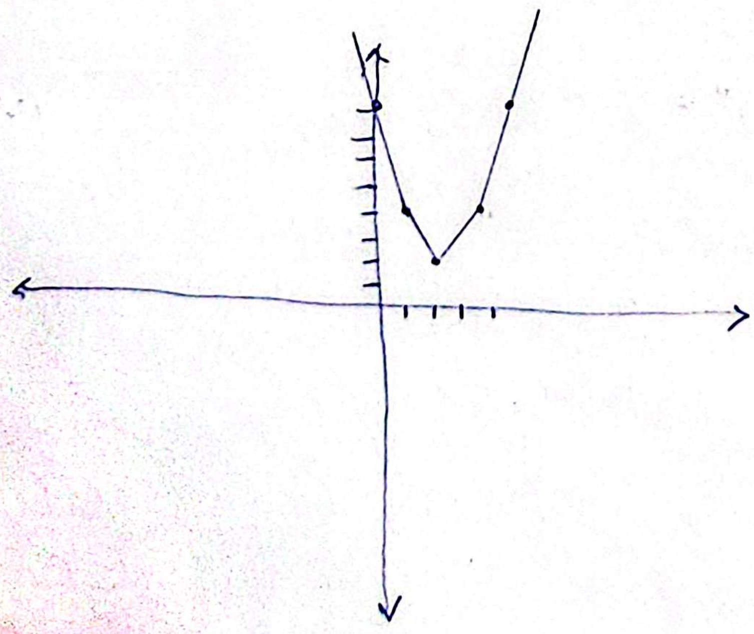
(۴)

$y = |x-1| + |x-3| + |2x-4|$

$x - 1 = 0 \rightarrow x = 1$
$x - 3 = 0 \rightarrow x = 3$
$2x - 4 = 0 \rightarrow x = 2$

(۲)

	1	2	3
$-x+1-x+3-2x+4$ $-4x+8$	$x-1-x+3-2x+4$ $-2x+6$	$x-1-x+3+2x-4$ $2x-2$	$x-1+x-3+2x-4$ $4x-8$
	(۴)	(۲)	(۴)



$y = 4x - 8 \xrightarrow{x=4} y = 8$

$y = -4x + 8 \xrightarrow{x=0} y = 8$

$\min y = 2$

(۵)

$y = |x| - 2|x+1|$ $\qquad x=0$, $x+1=0 \rightarrow x=-1$

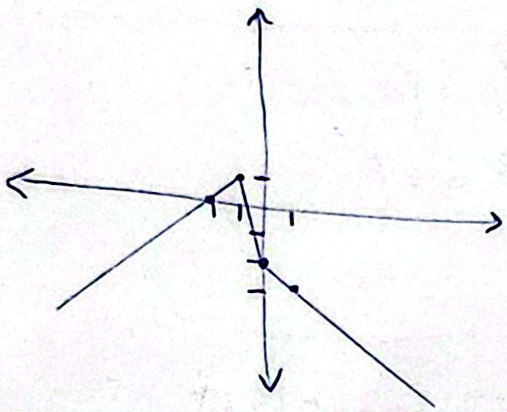
(۲)

$x<-1$	$-1<x<0$	$x>0$
$-x+2x+2$	$-x-2x-2$	$x-2x-2$
$x+2$	$-3x-2$	$-x-2$

At -1: (1) $\qquad$ At 0: (-2)

$y=-x-2 \xrightarrow{x=1} y=-3$

$y=x+2 \xrightarrow{x=-2} y=0$



$R_y = (-\infty, 1]$

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(۶)

$y = \dfrac{x^2+5x+m}{x+1} \rightarrow xy+y = x^2+5x+m \rightarrow x^2+(5-y)x+m-y=0$

(۲)

$x = \dfrac{y-5 \pm \sqrt{(5-y)^2-4m+4y}}{4} \Rightarrow y^2-10y+25-4m+4y \geq 0$

$\rightarrow y^2-6y+25-4m \geq 0 \xrightarrow{\Delta \leq 0} 36-4(25-4m) \leq 0$

$\rightarrow 36-100+16m \leq 0 \rightarrow 16m \leq 64 \rightarrow m \leq 4$

۴ نمی‌تواند باشد پس به ازای ۳ عدد طبیعی ← $\{1,2,3\}$
دامنه عوض می‌شود

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(۷)

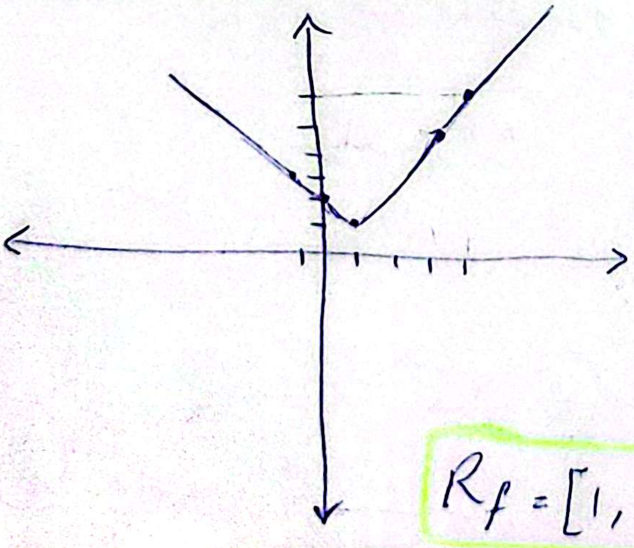
$y=x+2 \xrightarrow{x=3} y=5$
$\quad \xrightarrow{x=4} y=6$

(۲)

$y=x^2-2x+2 \xrightarrow{x=0} y=2$

$y=(x^2-2x+1)=(x-1)^2+1 \rightarrow S\left|\begin{matrix}1\\1\end{matrix}\right.$

$y=|x|+2 \xrightarrow{x=0} y=2$
$\quad \xrightarrow{x=-1} y=3$

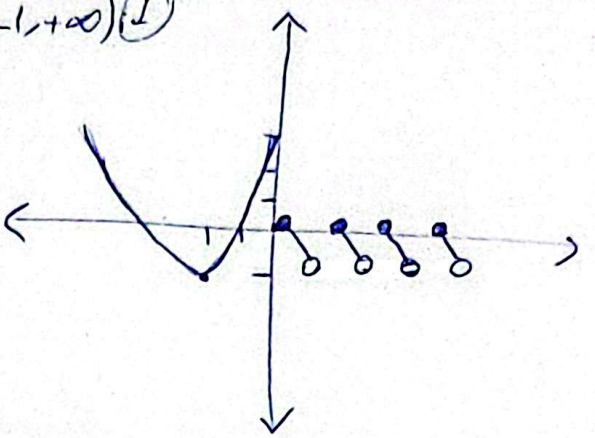


$R_f = [1, +\infty)$

$x^2+4x+4 \to 5 \begin{cases} -2 \\ -1 \end{cases} \to R=[-1,+\infty)$ (I) (۸)

(۲)

$[2x]-2x \to R=(-1,0]$ (II)



$R_f = (I) \cup (II) = [-1,+\infty)$

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(۹)

$2x+6 \geq 0 \to x \geq \frac{-6}{2} \to b=\frac{-6}{2}$

(۲)

$y_{max} \to \sqrt{2x+6}$ min $\to \sqrt{2x+6}=0 \to x=\frac{-6}{2}, y=a$

$y=a+1-0 \xrightarrow{y=a} a=a+1 \to a=4$

$a \times b = \frac{-6}{2} \times 4 = -12$

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(۱۰)

$f(x)=2\sqrt{x+1}+4\sqrt{1-x}$ $\quad \begin{matrix} x+1 \geq 0 \to x \geq -1 \\ 1-x \geq 0 \to x \leq 1 \end{matrix} \Big\} x=[-1,1]$

$f_{(1)}=2\sqrt{2}$ $\quad f_{(1)}+g_{(1)}=2\sqrt{2+2\sqrt{1-(1)^2}}=6\sqrt{2}$

$2\sqrt{2}+g_{(1)}=6\sqrt{2} \to g_{(1)}=\sqrt{2}$

(۲)

$f_{(-1)}=4\sqrt{2}$ $\quad f_{(-1)}+g_{(-1)}=2\sqrt{2+2\sqrt{1-(-1)^2}}=6\sqrt{2}$

$4\sqrt{2}+g_{(-1)}=6\sqrt{2} \to g_{(-1)}=-\sqrt{2}$

$y=\frac{f(x)}{4}-\frac{g(x)}{6} \xleftarrow{x=1} y=\frac{2\sqrt{2}}{4}-\frac{\sqrt{2}}{6}=0$

$\xrightarrow{x=-1} y=\frac{4\sqrt{2}}{4}-\frac{-\sqrt{2}}{6}=\sqrt{2}$

$R_y=[0,\sqrt{2}]$