

1. $f(x) = \sqrt{1-x^2}$ $g = \{(-1,1)(0,2)(1,2)\}$
 $g \cap f = \{(-1,1)(0,2)(1,2)\} \Rightarrow R_{fg} = \{1,2,2\}$

2. $f(n) = n-1$ $D_f = [1, +\infty)$ $R_f \cup R_g = ?$
 $g(n) = \frac{1}{n}n+3$ $D_g = (-\infty, 3]$ $R_f \cup R_g = \mathbb{R} - (2, \omega)$
 $\uparrow$

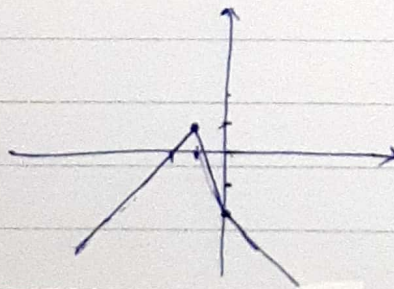
$f(x) \rightarrow R_f = [\omega, +\infty)$, $R_g = (-\infty, 2]$

3. $y = \frac{-x^2}{x} + x + 3$ $\frac{-x^2}{x} + x + 3 > \frac{3}{x} \rightarrow -\frac{x^2}{x} + x + \frac{3}{x} > 0 \rightarrow x^2 - x - 3 < 0$
 $\rightarrow (x-3)(x+1) < 0 \rightarrow x = (-1, 3) \Rightarrow \sqrt{b-a} = 4$

4. $y = |x-1| + |x-3| + |x-5|$ $y_{\min} = 4$
 $\begin{cases} -2x+4 & ; x \leq 1 \\ -x+4 & ; 1 < x \leq 3 \\ x-2 & ; 3 < x \leq 5 \\ 2x-4 & ; x > 5 \end{cases}$

5. $y = |x-1| - 2|x+1|$

$\begin{cases} x+2 & ; x \leq -1 \\ -3x-2 & ; -1 < x \leq 0 \\ -x-2 & ; x > 0 \end{cases}$

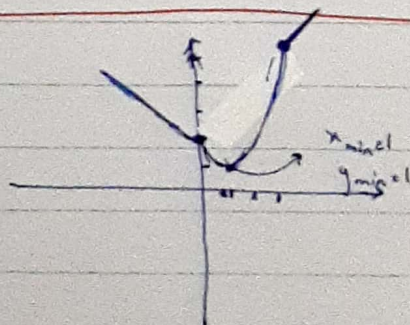


$\Rightarrow R_f = (-\infty, 1]$

6. $y = \frac{x^2 + 2x + m}{x+1} \rightarrow y(x+y) = x^2 + 2x + m \rightarrow x^2 + 2x + m - y(x+y) = 0 \rightarrow x = \frac{y-2 \pm \sqrt{(y-2)^2 - 4m-4y}}{2}$

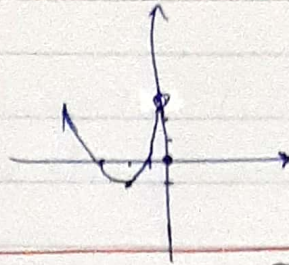
$\Rightarrow 4(y-2)^2 - 4m - 4y \geq 0 \Rightarrow 4y^2 + 14m - 10y \geq 0 \Rightarrow m \geq \frac{5}{7}y$

7. $f(n) = \begin{cases} n+1 & ; n \geq 3 \\ n^2 - 2n + 1 & ; 0 \leq n < 3 \\ |x| + 1 & ; n \leq 0 \end{cases}$



$R_f = [0, +\infty)$

$$8. f(x) = \begin{cases} x^2 + 5x + 4 - (x+4)(x+1) & ; x \leq 0 \\ \lfloor x \rfloor - x & ; x > 0 \\ -1 < \lfloor x \rfloor - x \leq 0 \end{cases}$$



$$\Rightarrow R_f = [-1, \infty)$$

$$9. y = a + 1 - \sqrt{x+4} \quad D_f = [b, +\infty) \quad R_f = (-\infty, a]$$

$$D_f: x+4 \geq 0 \Rightarrow x \geq -4 \Rightarrow b = -4 \quad a + 1 - \sqrt{x+4} = a \Rightarrow a = 1 \Rightarrow ab = -4$$

$$10. f(x) = 2\sqrt{x+1} + 3\sqrt{1-x} \quad f(x) + g(x) = 2\sqrt{x+1} + 3\sqrt{1-x} \quad D_f = D_g = [-1, 1]$$

$$D_f = [-1, 1]$$

$$\begin{cases} f(-1) + g(-1) = 2\sqrt{-1+1} + 3\sqrt{1-(-1)} = 0 + 3\sqrt{2} \Rightarrow g(-1) = 3\sqrt{2} \\ f(0) + g(0) = 2\sqrt{0+1} + 3\sqrt{1-0} = 2 + 3 = 5 \Rightarrow g(0) = 3 \\ f(1) + g(1) = 2\sqrt{1+1} + 3\sqrt{1-1} = 2\sqrt{2} + 0 \Rightarrow g(1) = 2\sqrt{2} \end{cases}$$

$$\Rightarrow \frac{f(x) - 3g(x)}{4} = \frac{f(x) - 3g(x)}{4} = \begin{cases} x = -1 \rightarrow \sqrt{2} \\ x = 0 \rightarrow 1 \\ x = 1 \rightarrow 0 \end{cases} \Rightarrow R_g = [0, \sqrt{2}]$$