۱-

$g=\{(-1,1)(0,4)(2,0)(1,2)\}$

$f=\{(-1,0)(0,1)\cancel{(2,\ )}(1,0)\}$

$\Rightarrow 2g-3f=\{(-1,2)\ (0,5)(1,4)\} \rightarrow 2+5+4=11$

۲-

$f(3)=2\times 3-1=5 \rightarrow R_f=[5,+\infty)$

$g(3)=\frac{1}{3}\times 3+3=4 \rightarrow R_g=(-\infty,4]$

$\overset{\cup}{\Longrightarrow} (-\infty,4]\cup[5,+\infty) = \mathbb{R}-(4,5)$

۳-

$-\frac{x^2}{2}+x+3>\frac{3}{2}$

$-x^2+2x+6>3 \Rightarrow x^2-2x-3<0$

$-\frac{c}{a}=3 \quad -1$

$-\infty$	-1		3	$+\infty$
$+$	0	$-$	0	$+$

$\Rightarrow a=-1,\ b=3 \rightarrow \sqrt{3-(-1)}=2$

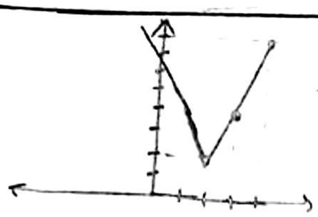
۴-

$x=1 \rightarrow y=4$

$x=2 \rightarrow y=2$

$x=3 \rightarrow y=4$

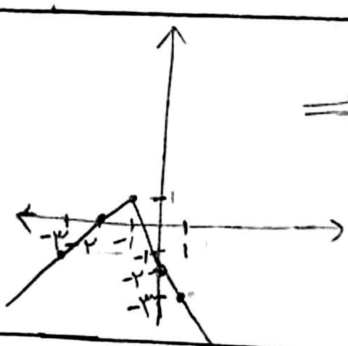
$x=4 \rightarrow y=8$



$\rightarrow$ کمترین مقدار $=2$

۵-

$$\begin{cases} -x-2 & ; x>0 \\ -3x-2 & ; -1\leq x\leq 0 \\ x+2 & ; x<-1 \end{cases}$$



$\Longrightarrow R=(-\infty,1]$

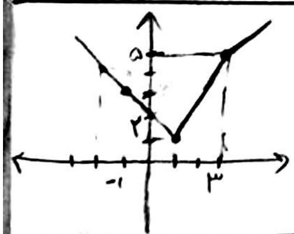
۶-

$yx+y=x^2+5x+m \Rightarrow x^2+(5-y)x+m-y \rightarrow x=\frac{-b\pm\sqrt{\Delta}}{2a} \xrightarrow{\Delta\geq 0} (5-y)^2-4(m-y)\geq 0$

$\Rightarrow y^2-10y+4y-4m+25\geq 0 \xrightarrow{\Delta\leq 0} 36-100m+16\leq 0 \longrightarrow m\leq 4 \rightarrow$ اما باید دانست که به ازای $m=4$ برد تابع $\mathbb{R}$ نمی‌شود و $x\neq -1$ باشد

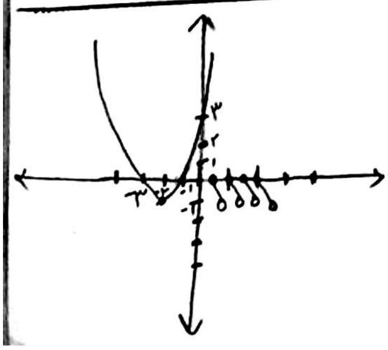
$\Rightarrow$ اعداد طبیعی حاصل $=\{1,2,3\}$

۷-



$\Rightarrow R=[1,+\infty)$

۸-



برای براکت $\Rightarrow 1\leq 2x<2 \rightarrow 1-2x$

$2\leq 2x<3 \rightarrow 2-x$

$\Longrightarrow (-1,0]$

$, R=[-1,+\infty)$

★ البته می‌شد بدون شکل هم رفت

-۹

$$2x+3 \geqslant 0 \rightarrow x \geqslant -\frac{3}{2} \Rightarrow b = -\frac{3}{2}$$

$$5 = a+1-\sqrt{2\left(-\frac{3}{2}\right)+3} \longrightarrow a = 4$$

$$\longrightarrow a \cdot b = 4 \times \left(-\frac{3}{2}\right) = \boxed{-6}$$

-۱۰

$$\left.\begin{array}{l} x+1 \geqslant 0 \rightarrow x \geqslant -1 \\ 1-x \geqslant 0 \rightarrow x \leqslant 1 \end{array}\right\} \longrightarrow D_f = D_g = [-1,1]$$

$$3\sqrt{\underbrace{2}_{1+1}+2\sqrt{\underbrace{1-x^2}_{(1-x)(1+x)}}} \xrightarrow{\text{با تشکیل مربع}} 3\sqrt{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2} = 3\sqrt{1-x}+3\sqrt{1+x}$$

$$g(x)+\underbrace{f(x)}_{2\sqrt{x+1}+4\sqrt{1-x}} = 3\sqrt{1-x}+3\sqrt{1+x} \Rightarrow g(x) = \sqrt{x+1}-\sqrt{1-x}$$

$$\frac{f(x)-2g(x)}{6} = \frac{2\sqrt{x+1}+4\sqrt{1-x}-2\left(\sqrt{x+1}-\sqrt{1-x}\right)}{6} = \sqrt{1-x} \Rightarrow R = [0, \sqrt{2}]$$