

$$f(x) = \sqrt{1-x^2}$$

$$f(-1) = \sqrt{1-(-1)^2} = 0$$

$$f(0) = \sqrt{1-0^2} = 1$$

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$$g = \{(-1, 1), (0, 4), (1, 0)\}$$

$$\Rightarrow y - xf = \{(-1, 2), (0, 5), (1, 4)\}$$

$$R = \{2, 4, 5\} \rightarrow 2+4+5 = 11$$

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$$\left. \begin{array}{l} f(x) = x^{n-1} \\ D_f = [3, +\infty) \end{array} \right\} \xrightarrow{\text{اجابیزای}} R_f = [5, +\infty)$$

$$\left. \begin{array}{l} g(x) = \frac{1}{x}x + 3 \\ D_g = (-\infty, 3] \end{array} \right\} \xrightarrow{\text{اجابیزای}} R_g = (-\infty, 4]$$

$$R_{f \cup g} = R_f \cup R_g = [12 - (4, 5)]$$

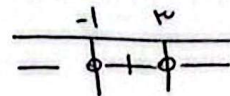
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$$y = \frac{1}{x}x^2 + x + 3$$

$$\frac{1}{x}x^2 + x + 3 > \frac{3}{x} \rightarrow \frac{1}{x}x + x + \frac{3}{x} > 0 \rightarrow \frac{1}{x}(x+1)(x-3) > 0$$

$$\rightarrow (a, b) = (-1, 3) \rightarrow \begin{array}{l} a = -1 \\ b = 3 \end{array}$$

$$\sqrt{b-a} = \sqrt{3-(-1)} = \sqrt{4} = 2$$

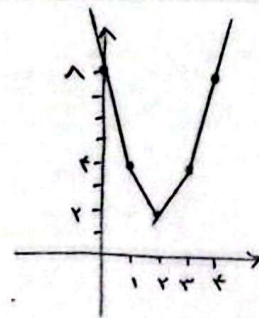


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$$y = |x-1| + |x-3| + |x-5|$$

Interval	Expression	Graph
$x < 1$	$1-x+3-x+5-x = -3x+9$	$(0, 9)$
$1 \leq x < 3$	$x-1+3-x+5-x = -x+7$	$(1, 6)$
$3 \leq x < 5$	$x-1+x-3+5-x = 1$	$(3, 1)$
$x \geq 5$	$x-1+x-3+x-5 = x-9$	$(5, 6)$

$$\Rightarrow \text{Minimum value} \Rightarrow R_f = [1, +\infty) \rightarrow \min = 1$$

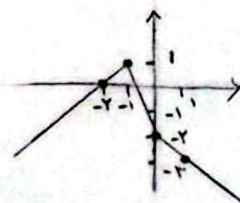


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$$y = |x-1| - 2|x+1|$$

Interval	Expression	Graph
$x < -1$	$-x+1-2(-x-1) = x+3$	$(-3, 0)$
$-1 \leq x < 1$	$-x+1-2(x+1) = -3x-1$	$(1, -3)$
$x \geq 1$	$x-1-2(x+1) = -x-3$	

$$\Rightarrow \text{Minimum value} \Rightarrow R_f = (-\infty, -1]$$



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$$y = \frac{n^2 + an + m}{n+1} \rightarrow y_{n+1} = \frac{n^2 + an + m}{n+1} \rightarrow n^2 + (a-y)n + (m-y) = 0$$

$\Delta \geq 0$ (برای $n \in \mathbb{R}$ جواب داشته باشد) $\rightarrow \Delta \geq 0$

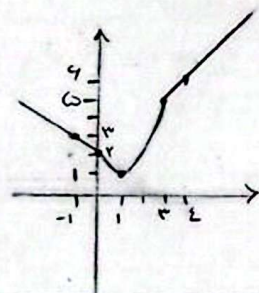
$$\Delta = (a-y)^2 - 4(m-y) \geq 0 \rightarrow y^2 - 4y + a^2 - 4m \geq 0 \rightarrow \Delta_{min} = 4 - 4m \geq 0 \rightarrow m \leq 1$$

اگر $m = 1$ باشد تابع خطی است و برای $n \in \mathbb{R}$ جواب دارد $\rightarrow m \neq 1$
 $\Rightarrow m \in \{1, 2, 3\}$
 و برای $n \in \mathbb{R}$ جواب دارد

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$$f(n) = \begin{cases} n+2 & ; n \geq 3 \rightarrow R_1 = [3, +\infty) \\ n^2 - 2n + 2 & ; 0 \leq n < 3 \rightarrow R_2 = [1, 5) \\ |n| + 2 & ; n < 0 \rightarrow R_3 = (-2, +\infty) \end{cases}$$

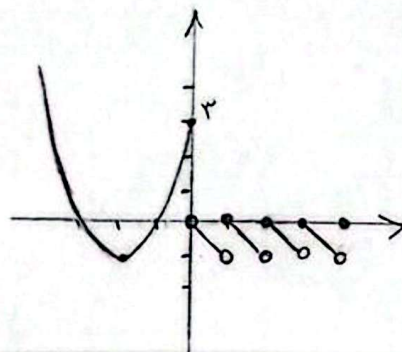
$$\Rightarrow R = R_1 \cup R_2 \cup R_3 = [1, +\infty)$$



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$$f(n) = \begin{cases} n^2 + 4n + 3 & ; n \leq 0 \rightarrow R_1 = [-1, +\infty) \\ [2n] \cdot 2n & ; n > 0 \rightarrow R_2 = [-1, 0] \end{cases}$$

$$\Rightarrow R = R_1 \cup R_2 = [-1, +\infty)$$



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$$y = a + 1 - \sqrt{2n+3}$$

$$D_f = [b, +\infty)$$

$$R_f = (-\infty, 5]$$

$$D \Rightarrow 2n+3 \geq 0 \rightarrow n \geq -\frac{3}{2} \rightarrow [-\frac{3}{2}, +\infty) \rightarrow b = -\frac{3}{2}$$

$$a+1 + \sqrt{0} = 5 \rightarrow a = 4$$

$$|ab| = \frac{3}{2} \times 4 = 6$$

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$$f(n) = 2\sqrt{n+1} + 4\sqrt{1-n}$$

$$\frac{f(n)+g(n)}{S(n)} = 2\sqrt{2+2\sqrt{1-n^2}} \rightarrow g(n) = S(n) - f(n)$$

$$y = \frac{f(n)}{4} - \frac{g(n)}{4} = \frac{f(n)}{4} - \frac{S(n)}{4} + \frac{f(n)}{4} = \frac{2\sqrt{2+2\sqrt{1-n^2}}}{4} - \frac{2\sqrt{2+2\sqrt{1-n^2}}}{4}$$

$$+ \frac{2\sqrt{n+1} + 4\sqrt{1-n}}{4} \Rightarrow \sqrt{n+1} + \sqrt{1-n} - \sqrt{2+2\sqrt{1-n^2}} \rightarrow D_y = \begin{cases} n > -1 \\ n < 1 \end{cases} \Rightarrow D = [-1, 1]$$

$$g(-1) = \sqrt{2} \quad g(1) = 0 \quad \Rightarrow R = [0, \sqrt{2}]$$

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