شهرزاد شوشتری　　کلاس ۸　　۲۰

۱)　۳

$f(x) = \sqrt{1-x^2} \to 1-x^2 \geq 0 \to x^2 \leq 1 \to -1 \leq x \leq 1$ $\begin{cases} x=1 \to 0 \\ x=0 \to 1 \\ x=-1 \to 0 \end{cases}$

$2g - 3f = \{(-1,2)(0,5)(1,4)\}$　　$R = \{2,5,4\}$　　$5+4+2=11$

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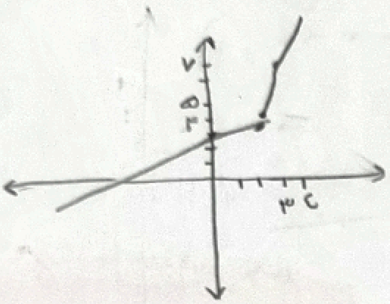
۲)　۳



$f(x) = 2x-1 \to x=3 \to 6-1=5$

$2x-1 \to x=4 \to 8-1=7$ $\Big\}$ $R_f = [5,+\infty)$

$g(x) = \frac{1}{3}x + 3 \to x=3 \to 1+3=4$

$x=0 \to 0+3=3$　　$R_g = (-\infty, 4]$

$R_f \cup R_g = (-\infty,4] \cup [5,+\infty)$

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۳)　۳

$-\frac{x^2}{2} + x + 3 \geq \frac{3}{2} \xrightarrow{\times 2} -x^2 + 2x + 6 \geq 3 \to -x^2 + 2x + 3 \geq 0 \to$

$x^2 + 2x - 3 = (x+3)(x-1)$　　$\begin{array}{c|c|c} & -1 & 3 \\ + & - & + \end{array} \to a = -1$

$\sqrt{3-(-1)} = \sqrt{4} = 2$

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۴)　۳

$y = |x-1| + |x-3| + |2x-4|$

$\downarrow \quad \downarrow \quad \downarrow$

$1 \quad 3 \quad 2$

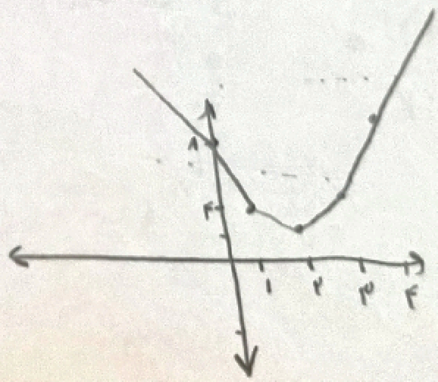
$x=0 \to |0-1| + |0-3| + |0-4| = 8$　　$\begin{array}{c|c} 0 \\ 8 \end{array}$

$x=1 \to |1-1| + |1-3| + |2-4| = 4$　　$\begin{array}{c|c} 1 \\ 4 \end{array}$

$x=2 \to |2-1| + |2-3| + |4-4| = 2$　　$\begin{array}{c|c} 2 \\ 2 \end{array}$

$x=3 \to |3-1| + |3-3| + |6-4| = 4$　　$\begin{array}{c|c} 3 \\ 4 \end{array}$

$x=4 \to |4-1| + |4-3| + |8-4| = 8$　　$\begin{array}{c|c} 4 \\ 8 \end{array}$



کمترین مقدار = ۲

شهرزاد شونسی

۵) ۲

$y = |x| - 2|x+1|$

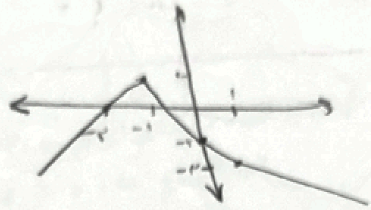
$0 \qquad -1$

$x = 1 \to |1| - 2|2| = -3$

$x = 0 \to |0| - 2|0+1| = -2$

$x = -1 \to |-1| - 2|0| = 1$

$x = -2 \to |-2| - 2|-2+1| = 0$



$R_y = (-\infty, 1]$

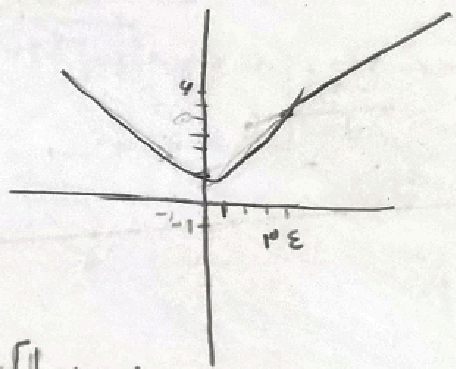
۶) ۲

$yx + y = x^2 + 5x + m \to x^2 + x(5-y) + m - y$

$(5-y)^2 - 4(m-y) \to 25 + y^2 - 10y - 4m + 4y \Rightarrow y^2 - 6y + 25 - 4m$

$36 - 4(25 - 4m) < 0 \to 36 - 100 + 16m < 0 \to 16m < 64 \to m < 4 \to m = 1, 2, 3$

۷) ۳



$x \geq 3 \to x + 2$

$x = 3 \to 3 + 2 = 5$

$x = 4 \to 4 + 2 = 6$

$x^2 - 2x + 2 \quad 0 \leq x < 3$

$x = 3 \to 9 - 6 + 2 = 5$

$x = 0 \to 0 - 0 + 2 = 2$

$x < 0 \to |x| + 2$

$x = 0 \to 0 + 2 = 2$

$x = -1 \to |-1| + 2 = 3$

$R_f = [1, +\infty)$

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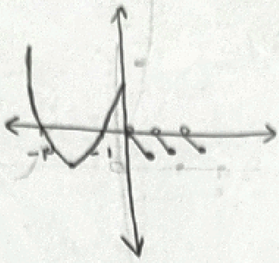
۸)

۳

$$f(x)=\begin{cases} x^2+4x+3 & x\leq 0 \\ [2x]-2x & x>0 \end{cases} \qquad (x+1)(x+3)$$



$y=x^2+4x+3=(x+1)(x+3)\to x=-1 ,\ x=-3$

$R_y=[-1,+\infty)$

$0\leq [2x]-2x<-1$

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۹)

$y=a+2\sqrt{2x+3}$

$D_y: 2x+3\geq 0\to 2x\geq -3\to x\geq -\frac{3}{2} \qquad b=-\frac{3}{2}$

۳

$x=-\frac{3}{2}\to a+1-\sqrt{\cancel{2\times\frac{-3}{2}+3}}=5\to a+1=5\to a=4$

$ab=-\frac{3}{\cancel{2}}\times \overset{2}{\cancel{4}}=-6$

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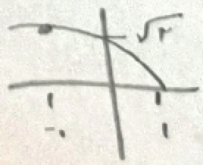
۱۰)

۳

$f(x)+g(x)=3\sqrt{2+2\sqrt{1-x^2}} \qquad f(x)=2\sqrt{x+1}+4\sqrt{1-x}$

$f(x)+g(x)=\left(3\sqrt{2+2\sqrt{1-x^2}}\right)^2\to 9\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\to 3\sqrt{1-x}+\sqrt{1+x}$

$g(x)=\sqrt{1+x}-\sqrt{1-x} \qquad \frac{f(x)}{6}-\frac{g(x)}{6}=\sqrt{1-x}$



$R=[0,\sqrt{2}]$