

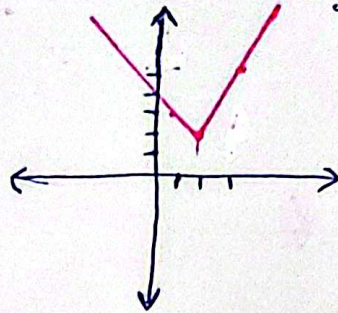
(1)

الف) $y = x + |2x - 1|$

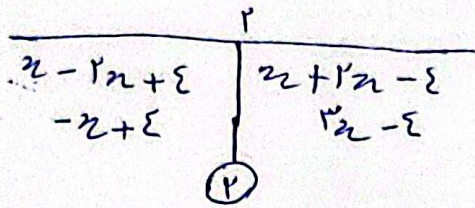
$2x - 1 = 0 \rightarrow x = \frac{1}{2}$

$y = 2x - 1 \xrightarrow{x=\frac{1}{2}} y = 0$

$y = -x + 1 \xrightarrow{x=1} y = 0$

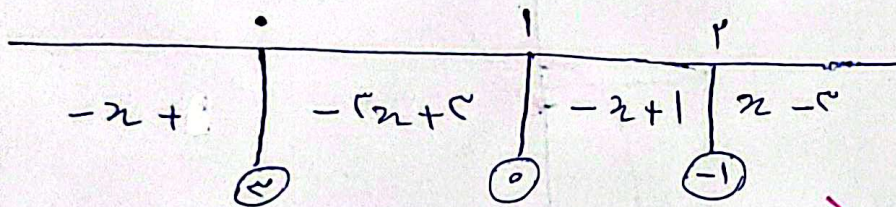


$R_y = [0, +\infty)$



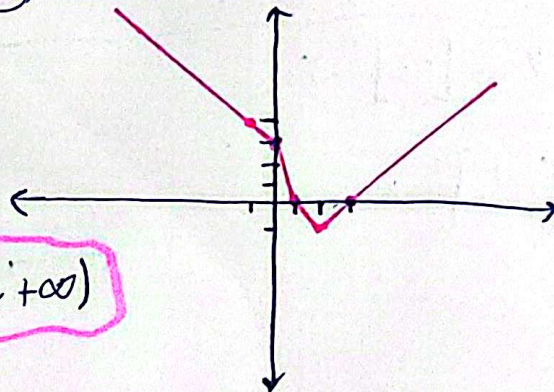
ب) $y = |x - 2| + |x - 1| - |x|$

$x - 2 = 0 \rightarrow x = 2$
 $x - 1 = 0 \rightarrow x = 1$
 $x = 0$



$y = x - 2 \xrightarrow{x=2} y = 0$

$y = -x + 2 \xrightarrow{x=-2} y = 4$

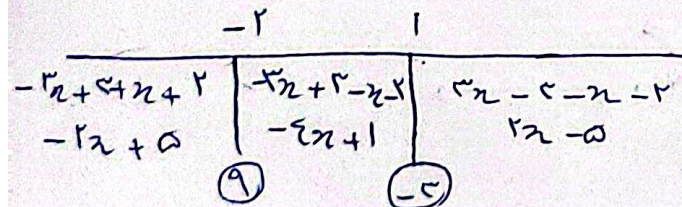


$R_y = [-1, +\infty)$

$|3x - 3| - |x + 2| = k$

$x + 2 = 0 \rightarrow x = -2$
 $3x - 3 = 0 \rightarrow x = 1$

(2)

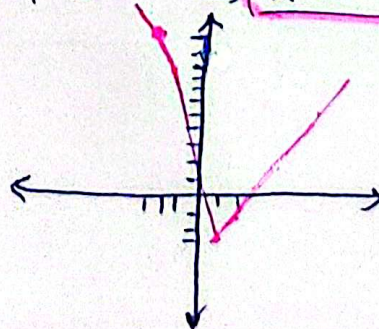


$\min \rightarrow x = 1$

$x = 1 \rightarrow |3 - 3| - |1 + 2| = |0| - |3| = -3 \rightarrow k = -3$

$y = 3x - 5 \xrightarrow{x=\frac{5}{3}} y = 0$

$y = -3x + 5 \xrightarrow{x=-\frac{5}{3}} y = 10$



$$\text{الف) } y = x + \frac{x}{|x|}$$

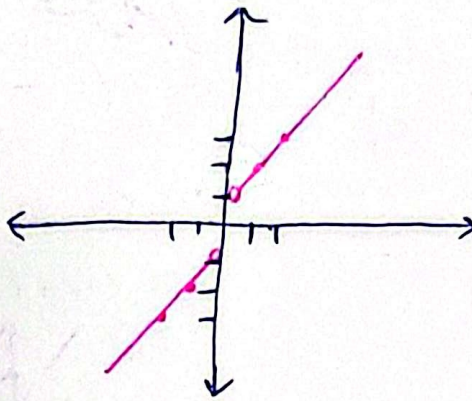
$$\frac{\circ}{x-1 \text{ } \& \text{ } x+1}$$

$$y = x+1 \xrightarrow{x=1} y=2$$

$$y = x-1 \xrightarrow{x=-1} y=-2$$

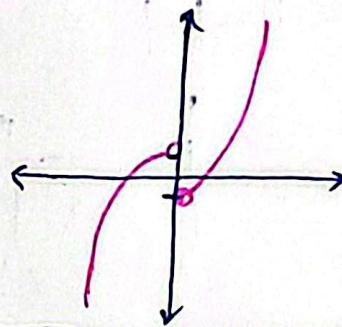
$$y = x+1 \xrightarrow{x=2} y=3$$

$$y = x-1 \xrightarrow{x=-2} y=-3$$



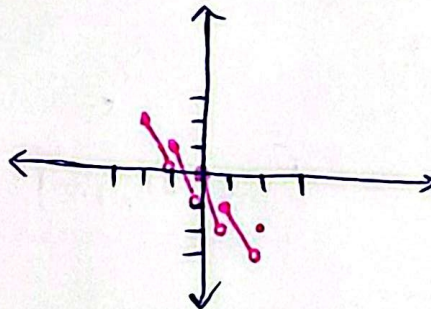
$$\text{ب) } y = x/|x| - \frac{x}{|x|}$$

$$\frac{\circ}{-x^2+1 \text{ } \& \text{ } x^2-1}$$



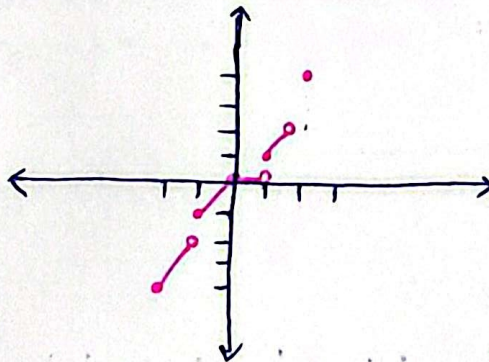
$$\text{الف) } y = [x] - 2x$$

$$[-1, 1]$$



$$\text{ب) } y = [x]/|x|$$

$$[-1, 1]$$



الف) $y = 3x - [3x] \rightarrow R_f = [0, 1)$

ب) $y = 2x - 2[x] \rightarrow 2(x - [x]) \rightarrow R_f = [0, 2)$

ج) $y = x - a\left[\frac{x}{a}\right] \rightarrow a\left(\frac{x}{a} - \left[\frac{x}{a}\right]\right) \rightarrow R_f = [0, a)$

د) $y = [x] - x \rightarrow (-1, 0]$

الف) $y = 3\sin x + 4\cos x \rightarrow -\sqrt{17} \leq 3\sin x + 4\cos x \leq \sqrt{17}$

$R_f = [-\sqrt{17}, \sqrt{17}]$

ب) $y = 5\sin x - 3\cos x \rightarrow -\sqrt{19+9} \leq 5\sin x - 3\cos x \leq \sqrt{19+9}$

$R_f = [-5, 5]$

ج) $y = [3\sin x + 4\cos x] \rightarrow -\sqrt{25} \leq 3\sin x + 4\cos x \leq \sqrt{25}$

$[-5, 5] \Rightarrow R_y = \{y \mid y \in \mathbb{Z}, -5 \leq y \leq 5\}$

د) $y = \sqrt{\cos x - \sin x} \rightarrow -\sqrt{2} \leq \cos x - \sin x \leq \sqrt{2}$

$\sqrt{[-2, 2]} \rightarrow R_y = [0, \sqrt{2}]$

الف) $y = \sin^2 x + \cos^2 x + 1$ ب) $y = 3\sin^2 x + \cos^2 x + 1$

$\frac{-b}{2a} \rightarrow \frac{-5}{2} \rightarrow y = \frac{-1}{2}$ $\frac{-b}{2a} = \frac{-3}{2} \rightarrow y = \frac{1}{2}$

$x = -1 \rightarrow y = 0$

$x = 1 \rightarrow y = 6$

$R_y = [0, 6]$

$x = -1 \rightarrow y = 1$

$x = 1 \rightarrow y = 5$

$R_y = \left[\frac{1}{2}, 5\right]$

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الف) $y = \sin^2 z + \epsilon \cos^2 z$

$$\frac{\sin^2 z + \cos^2 z}{1} + \frac{\epsilon \cos^2 z}{[0, \epsilon]} \rightarrow Ry = [1, \epsilon]$$

$0 \leq \cos^2 z \leq 1$

ب) $y = \frac{\epsilon \cot z}{1 + \cot^2 z}$

$$\frac{\frac{\epsilon \cos z}{\sin z}}{\frac{1}{\sin^2 z}} = \frac{\epsilon \cos z \sin^2 z}{\sin z} = \epsilon \sin z \cos z = \sin^2 z$$

$Ry = [-1, 1]$

الف) $y = \sin^4 z + \cos^4 z$

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$$1 - \epsilon \leq \sin^4 z + \cos^4 z \leq 1 \rightarrow \frac{1}{\epsilon} \leq \sin^4 z + \cos^4 z \leq 1$$

$Ry = [\frac{1}{\epsilon}, 1]$

ب) $y = [\sin^8 z + \cos^8 z]$

$$1 - \epsilon \leq \sin^8 z + \cos^8 z \leq 1 \rightarrow \frac{1}{\epsilon} \leq \sin^8 z + \cos^8 z \leq 1$$

$Ry = \{0, 1\}$ چون برکت داریم

الف) $y = \frac{z^2 + \sqrt{z} - \epsilon}{z + \epsilon} = \frac{(z + \epsilon)(z - 1)}{(z + \epsilon)} = z - 1$

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$z \neq \pm \epsilon \rightarrow y \neq -1$

$Ry = R - \{-1\}$

$z \neq 1, y \neq 0$

$$ب) y = \frac{z^2 - 1}{z - 1} = \frac{(z - 1)(z^2 + z + 1)}{(z - 1)} = z^2 + z + 1$$

$Ry = [\frac{5}{4}, +\infty)$

