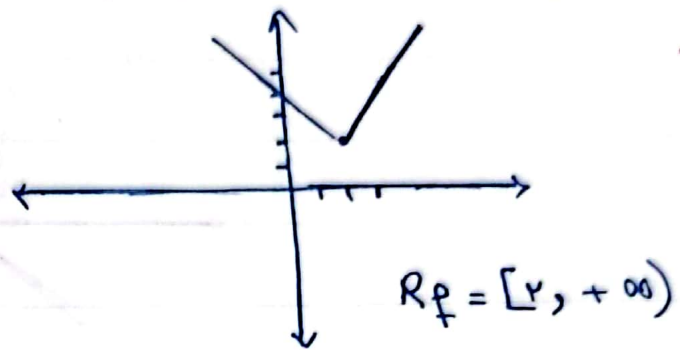


نرانه طهریان - تکیف شماری ۷ - یازدهم دفتر

۱-

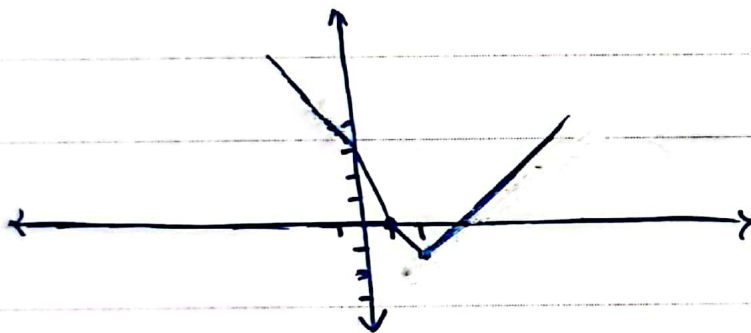
الف) $y = x + |2x - 4|$

$$\begin{array}{c|c} 2 & \\ \hline 2-2x+4 & x+2x-4 \\ \hline = -x+6 & = 3x-4 \end{array}$$



ب) $y = |x-2| + |x-1| - |x|$

$$\begin{array}{c|c|c|c} 0 & 1 & 2 & \\ \hline -x+2-x+1+x & -x+2-x+1-x & -x+2+x-1 & x-2+x-1-x \\ \hline = -x+3 & = -3x+3 & = -x+1 & = x-3 \end{array}$$



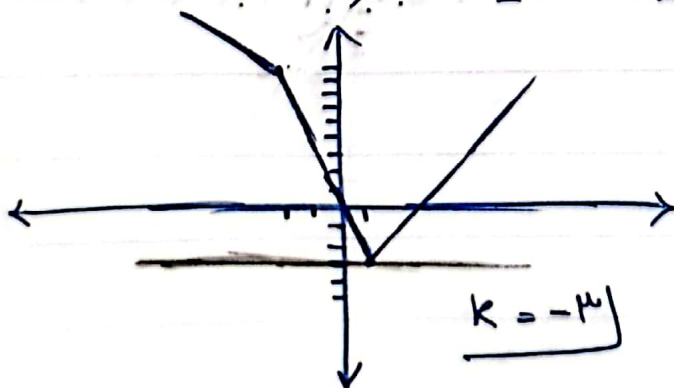
$R_f = [-1, +\infty)$

$y = |3x-3| - |x+2|$

$$\begin{array}{c|c|c} -2 & 1 & \\ \hline -3x+3+x & -3x+3-x & 3x-3-x-2 \\ \hline +2 & -2 & = 2x-5 \\ \hline = -2x+5 & = -4x+1 & \end{array}$$

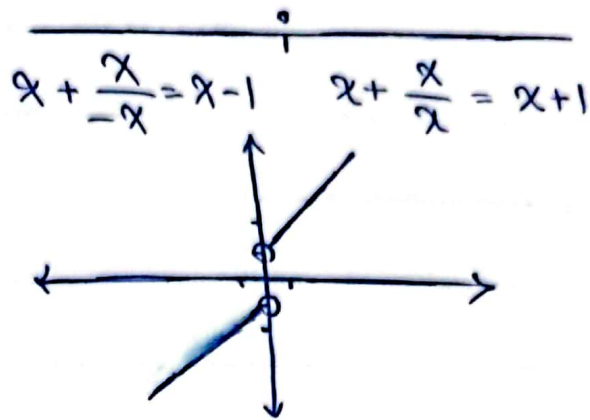
۲-

برای آنکه معادله فقط یک ریشه داشته باشد باید فقط در یک نقطه برابر با ۰ باشد :



الف) $y = x + \frac{x}{|x|}$

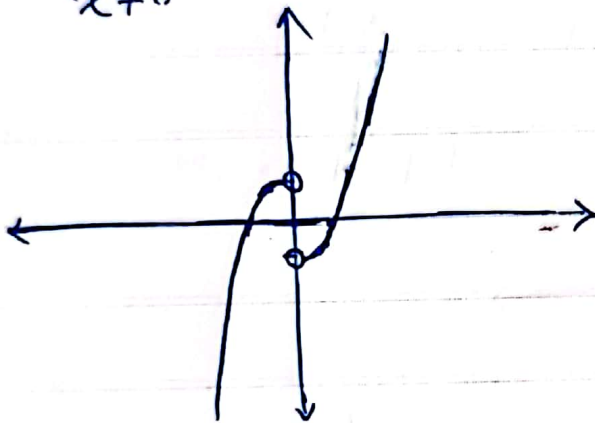
$x \neq 0$



ب) $y = x|x| - \frac{x}{|x|}$

$x \neq 0$

$-x^2 - \frac{x}{-x} = -x^2 + 1$
 $x^2 - \frac{x}{x} = x^2 - 1$



الف) $y = [x] - 2x$

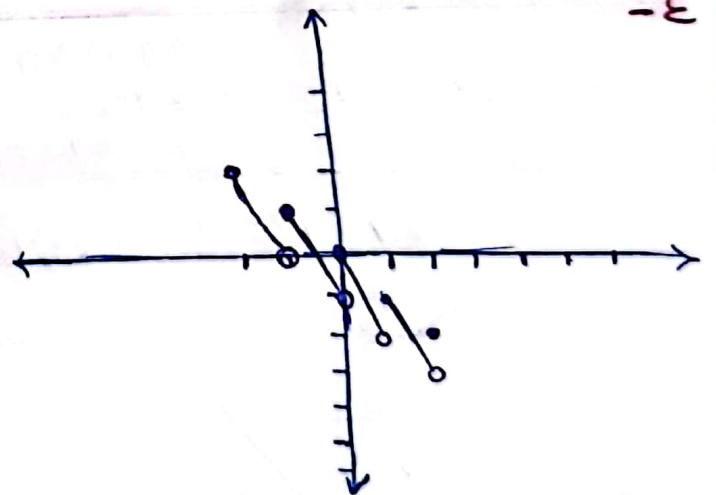
$x = 2 \rightarrow 2 - \varepsilon = -2$

$1 < x < 2 \rightarrow 1 - 2x$

$0 < x < 1 \rightarrow 0 - 2x = -2x$

$-1 < x < 0 \rightarrow -1 - 2x$

$-2 < x < -1 \rightarrow -2 - 2x$



ب) $y = [x]|x|$

$x = 2 \rightarrow 2$

$0 < x < 1 \rightarrow 0$

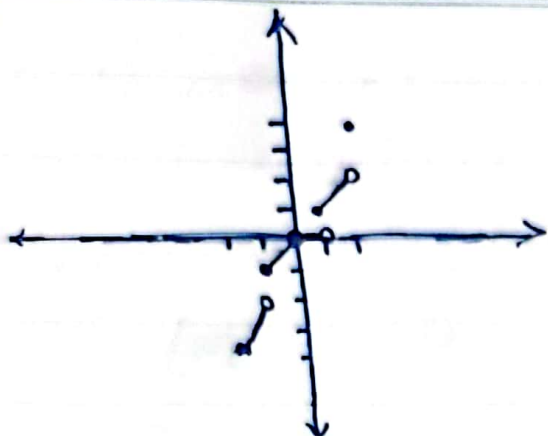
$-2 < x < -1 \rightarrow (-2)x(-x) = 2x$

$1 < x < 2 \rightarrow x$

$-1 < x < 0 \rightarrow -1 \times (-x) = x$

$= 2x$

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الف) $y = \mu x - [\mu x] \leadsto R_f = [0, 1)$

$0 \leq \mu x - [\mu x] < 1$

ب) $y = \varepsilon x - \kappa[x] = \kappa(x - [x])$

$0 \leq x - [x] < 1$

$x \xrightarrow{\kappa} 0 \leq \varepsilon x - \kappa[x] < \kappa \quad R_f = [0, \kappa)$

2) $y = x - \omega \left[\frac{x}{\omega} \right] = \omega \left(\frac{x}{\omega} - \left[\frac{x}{\omega} \right] \right)$

$0 \leq \frac{x}{\omega} - \left[\frac{x}{\omega} \right] < 1 \xrightarrow{\times \omega} 0 \leq x - \omega \left[\frac{x}{\omega} \right] < \omega \quad R_f = [0, \omega)$

3) $y = [x] - x = -(x - [x])$

$0 \leq x - [x] < 1 \xrightarrow{\times -1} -1 < [x] - x \leq 0 \quad R_f = (-1, 0]$

الف) $y = \mu \sin x + \nu \cos x \rightarrow -\sqrt{\mu^2 + \nu^2} \leq \mu \sin x + \nu \cos x \leq \sqrt{\mu^2 + \nu^2}$

$\Rightarrow -\sqrt{1\mu^2 + 1\nu^2} \leq \mu \sin x + \nu \cos x \leq \sqrt{1\mu^2 + 1\nu^2} \quad R_f = [-\sqrt{1\mu^2 + 1\nu^2}, \sqrt{1\mu^2 + 1\nu^2}]$

ب) $y = \kappa \sin x - \mu \cos x \rightarrow -\sqrt{\kappa^2 + (-\mu)^2} \leq \kappa \sin x - \mu \cos x \leq \sqrt{\kappa^2 + (-\mu)^2}$

$\Rightarrow -\sqrt{\kappa^2 + \mu^2} \leq \kappa \sin x - \mu \cos x \leq \sqrt{\kappa^2 + \mu^2}$

$R_f = [-\omega, \omega]$

$$ج) y = [r \sin x + \omega \cos x] \rightarrow -\sqrt{r^2 + \omega^2} \leq r \sin x + \omega \cos x \leq$$

$$\sqrt{r^2 + \omega^2} \Rightarrow -\sqrt{r^2 + \omega^2} \leq r \sin x + \omega \cos x \leq \sqrt{r^2 + \omega^2} \Rightarrow$$

$$-4 \leq [r \sin x + \omega \cos x] \leq \omega \quad R_f = [-4, \omega]$$

$$r\omega < r^2 < \omega^2 \Rightarrow \omega < \sqrt{r^2 + \omega^2} < 4$$

$$د) y = \sqrt{\cos x - r \sin x} \leadsto -\sqrt{1 + (-r)^2} \leq \cos x - r \sin x \leq$$

$$\sqrt{1 + (-r)^2} \Rightarrow -\sqrt{\omega} \leq \cos x - r \sin x \leq \sqrt{\omega} \Rightarrow 0 \leq \sqrt{\cos x - r \sin x} \leq \sqrt{\omega}$$

$$R_f = [0, \sqrt{\omega}]$$

$$الف) y = 8 \sin^2 x + 3 \sin x + 2$$

$$\sin x = -1 \rightarrow 1 - 3 + 2 = 0 \quad R_f = [0, 4]$$

$$\sin x = 1 \rightarrow 1 + 3 + 2 = 4$$

$$\sin x = \frac{-b}{a} = -\frac{3}{8} \rightarrow 9 - 9 + 2 = 2$$

$$ب) y = r \sin^2 x + 3 \sin x + 2$$

$$\sin x = -1 \rightarrow r - 3 + 2 = 1 \quad R_f = [1, 4]$$

$$\sin x = 1 \rightarrow r + 3 + 2 = 4$$

$$\sin x = \frac{-b}{a} = \frac{-3}{r} \rightarrow r \times \frac{9}{r} - \frac{9}{r} + 2 = 2$$

$$الف) y = 8 \sin^2 x + k \cos^2 x = 8 \sin^2 x + k(1 - \sin^2 x) = -k$$

$$8 \sin^2 x - k \sin^2 x + k = -k \sin^2 x + k$$

$$0 \leq \sin^2 x \leq 1 \xrightarrow{x \rightarrow 0} -k \leq -k \sin^2 x \leq 0 \xrightarrow{x \rightarrow 1} 1 - k \sin^2 x + k \leq k$$

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$$R_f = [1, k]$$

$$\text{ب) } y = \frac{y \cot x}{1 + \cot x} = \frac{\frac{y \cos x}{\sin x}}{\frac{1}{\sin x}} = \frac{y \cos x \sin x}{\sin x} = y \sin x \cos x = \sin 2x$$

$$-1 < \sin 2x < 1 \quad R_f = [-1, 1]$$

$$\text{الف) } y = \sin^y x + \cos^y x \xrightarrow{y^{1-k} \leq \sin^k x + \cos^k x < 1}$$

$$y^{1-\frac{1}{\epsilon}} \leq \sin^{\frac{1}{\epsilon}} x + \cos^{\frac{1}{\epsilon}} x < 1 \Rightarrow R_f = \left[\frac{1}{\epsilon}, 1\right]$$

$$\downarrow$$

$$y - \frac{1}{\epsilon} = \frac{1}{\epsilon}$$

$$\text{ب) } y = [\sin^{\frac{1}{\epsilon}} x + \cos^{\frac{1}{\epsilon}} x] \Rightarrow \frac{1-\epsilon}{\epsilon} \leq \sin^{\frac{1}{\epsilon}} x + \cos^{\frac{1}{\epsilon}} x < 1$$

$$y - \frac{1}{\epsilon} = \frac{1}{\epsilon}$$

$$\Rightarrow [\sin^{\frac{1}{\epsilon}} x + \cos^{\frac{1}{\epsilon}} x] \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right. \quad R_f = \{0, 1\}$$

$$\text{الف) } y = \frac{yx^y + yx - \epsilon}{x + \epsilon} = yx - 1 \quad -10$$

$$\frac{yx^y + yx - \epsilon}{x + \epsilon} \xrightarrow{x \neq -\epsilon} y \neq yx(-\epsilon) - 1 = -9$$

$$\begin{array}{r} yx^y + yx - \epsilon \quad | \quad x + \epsilon \\ -yx^y - \epsilon x \quad | \quad yx - 1 \\ \hline -x - \epsilon \\ +x + \epsilon \\ \hline 0 \end{array}$$

$$R_f = \mathbb{R} - \{-9\}$$

$$\text{ب) } y = \frac{x^y - 1}{x - 1} = \frac{(x-1)(x^y + x + 1)}{(x-1)} = x^y + x + 1$$

$$\xrightarrow{x \neq 1} y \neq 1 + 1 + 1 = 3 \quad R_f = \mathbb{R} - \{3\}$$