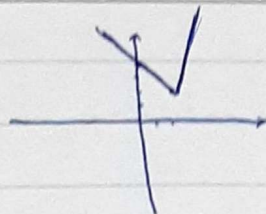


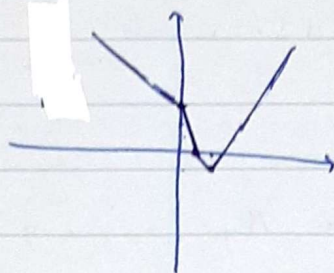
۱- $y = x + |2x - 2|$ $\rightarrow R_f = [1, +\infty)$



$$\begin{cases} 2x - 2 & ; x \geq 1 \\ -x + 2 & ; x \leq 1 \end{cases}$$

۲- $y = |x - 2| + |x - 1| - |x|$

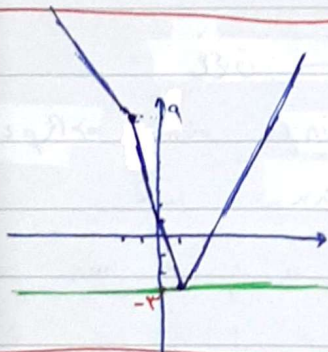
$$\begin{cases} -x + 3 & ; x \leq 0 \\ -2x + 3 & ; 0 \leq x \leq 1 \\ -x + 1 & ; 1 \leq x \leq 2 \\ x - 3 & ; x \geq 2 \end{cases}$$



$\rightarrow R_f = [-1, +\infty)$

۳- $|3x - 3| - |x + 2| = k$

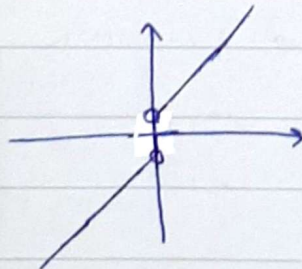
$$\begin{cases} -2x + 5 & ; x \leq -2 \\ -x + 1 & ; -2 \leq x \leq 1 \\ 2x - 5 & ; x \geq 1 \end{cases}$$



$\rightarrow A = k$ $\rightarrow k = -3$

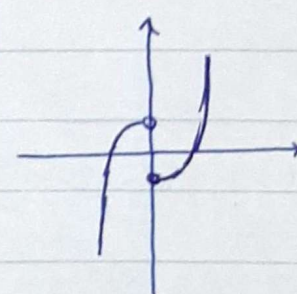
۴- $y = x + \frac{x}{|x|}$

$$\begin{cases} x + 1 & ; x > 0 \\ x - 1 & ; x < 0 \end{cases}$$



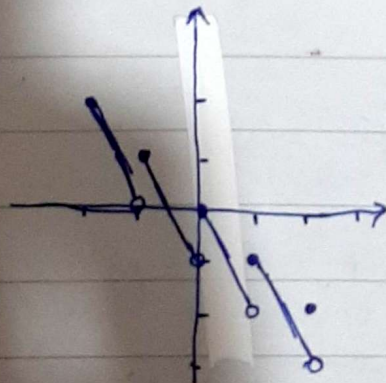
۵- $y = x|x| - \frac{x}{|x|}$

$$\begin{cases} x^2 - 1 & ; x > 0 \\ -x^2 + 1 & ; x < 0 \end{cases}$$

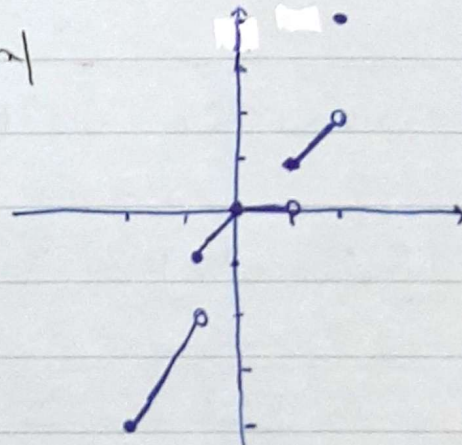


۶- $y = [x] - 2x$

$[-2, 2]$



۷- $y = [x] |x|$



$$\omega - \text{ا}) y = \omega n - [\omega n] \Rightarrow R_f = [0, 1)$$

$$\text{ب}) y = \varepsilon n - \varepsilon [n] \Rightarrow y \in (\varepsilon(n - [n])) \Rightarrow R_f = [0, \varepsilon)$$

$$\text{ج}) y = n - a \left[\frac{n}{a} \right] \Rightarrow y = a \left(\frac{n}{a} - \left[\frac{n}{a} \right] \right) \Rightarrow R_f = [0, a)$$

$$\text{د}) y = [rn] - rn \Rightarrow y = rn - [rn] \Rightarrow R_f = (-1, 0]$$

$$\gamma - \text{ا}) y = r \sin n + r \cos n \Rightarrow R_f = [-\sqrt{r^2}, \sqrt{r^2}]$$

$$\text{ب}) y = r \sin n - r \cos n \Rightarrow R_f = [-a, a]$$

$$\text{ج}) y = \underbrace{r \sin n + a \cos n}_A \Rightarrow -\sqrt{r^2 + a^2} \leq A \leq \sqrt{r^2 + a^2} \Rightarrow R_f = \{-a, -\varepsilon, -r, -r, -1, 0, 1, r, \varepsilon, a\}$$

$$\text{د}) y = \sqrt{\cos n - r \sin n}$$

$$-\sqrt{a} \leq \cos n - r \sin n \leq \sqrt{a} \Rightarrow R_y = [0, \sqrt{a}]$$

$$\nu - \text{ا}) y = \sin^2 n + r \sin n + r$$

$$\begin{cases} \sin n = -\frac{r}{r} \Rightarrow -\sqrt{r^2} \\ \sin n = -1 \Rightarrow y = 0 \text{ min} \Rightarrow R_f = [0, y] \\ \sin n = 1 \Rightarrow y = r \text{ max} \end{cases}$$

$$\text{ب}) y = r \sin^2 n + r \sin n + r$$

$$\begin{cases} \sin n = -\frac{r}{r} \Rightarrow y = \frac{r}{r} \text{ min} \\ \sin n = -1 \Rightarrow y = 1 \\ \sin n = 1 \Rightarrow y = r \text{ max} \end{cases} \Rightarrow R_f = \left[\frac{r}{r}, r \right]$$

$$\lambda - \text{ا}) y = \sin^2 n + \varepsilon \cos^2 n = \underbrace{r \cos^2 n + 1}_{[0, r]} \Rightarrow R_f = [1, \varepsilon]$$

$$\text{ب}) y = \frac{r \cot n}{1 + \cot^2 n} = \frac{r \cos n}{\sin n} \times \sin^2 n = r \cos n \sin n = r \times \frac{1}{2} \times \sin 2n \Rightarrow R_f = [-1, 1]$$

$$\theta - \text{ا}) y = \sin^4 n + \cos^4 n \rightarrow R_f = \left[\frac{1}{2}, 1 \right]$$

$$\text{ب}) y = [\sin^4 n + \cos^4 n] \rightarrow \frac{r}{\frac{1}{2}} [\sin^4 n + \cos^4 n] \leq 1 \Rightarrow R_f = \{0, 1\}$$

$$\mu - \text{ا}) y = \frac{rn^2 + rn - \varepsilon}{n + \varepsilon} = \frac{(rn-1)(n+\varepsilon)}{(n+\varepsilon)} = rn-1 \rightarrow \begin{array}{c} \text{Graph of } y = rn-1 \text{ showing a line passing through } (0, -1) \text{ and } (1, r-1) \end{array} \Rightarrow R_f = \mathbb{R} - \{-1\}$$

$$\text{ب}) y = \frac{n^3 - 1}{n - 1} = \frac{(n^2 + n + 1)(n-1)}{(n-1)} = n^2 + n + 1 \rightarrow \begin{array}{c} \text{Graph of } y = n^2 + n + 1 \text{ showing a parabola opening upwards} \end{array} \rightarrow R_f = \mathbb{R}$$