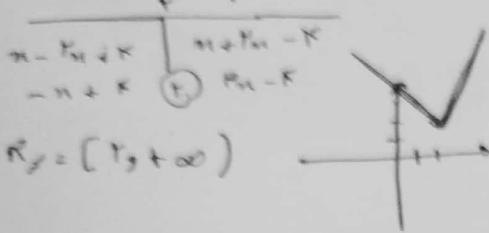


الف) $y = x + |x - 2|$



$R_f = [2, +\infty)$

ب) $y = |x - 2| + |x - 1| - |x|$

$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

$R_f = [-1, +\infty)$

$|x - 2| - |x + 1| = k$

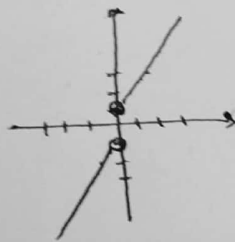
$\begin{array}{l} x < -1: -x + 2 - (-x - 1) = 3 \\ -1 < x < 2: -x + 2 - (x + 1) = 1 - 2x \\ x > 2: -x + 2 - (x + 1) = 1 - 2x \end{array}$

$\Rightarrow x = 1$

$|1 - 2| - |1 + 1| = -4 \rightarrow k = -4$

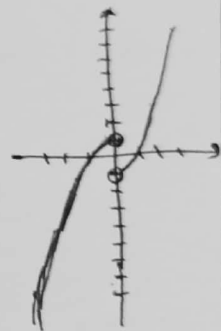
الف) $y = x + \frac{x}{|x|}$

$\begin{array}{l} x < 0: -1 \\ x > 0: 1 \end{array}$



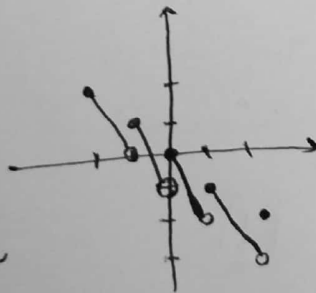
ب) $y = x|x| - \frac{x}{|x|}$

$\begin{array}{l} x < 0: -x^2 - 1 \\ x > 0: x^2 - 1 \end{array}$



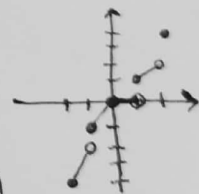
الف) $y = [x] - 2x$

$x = 2 \rightarrow y = -2$
 $1 \leq x < 2 \rightarrow -2x$
 $0 \leq x < 1 \rightarrow -2x$
 $-1 \leq x < 0 \rightarrow -2x - 1$
 $-2 \leq x < -1 \rightarrow -2x - 2$



ب) $y = [x] |x|$

$x = 2 \rightarrow y = 2$
 $1 \leq x < 2 \rightarrow |x|$
 $0 \leq x < 1 \rightarrow 0$
 $-1 \leq x < 0 \rightarrow -|x|$
 $-2 \leq x < -1 \rightarrow -2|x|$



الف) $y = 3x - [3x]$

$0 \leq 3x - [3x] < 1$

$R_f = [0, 1)$

ب) $3x - 3[x]$

$3(x - [x]) \rightarrow [0, 3)$

ج) $y = x - \omega\left[\frac{x}{\omega}\right]$

$\omega\left(\frac{x}{\omega}\right) \rightarrow \omega\left(\frac{x}{\omega} - \left[\frac{x}{\omega}\right]\right)$

$R_f = [0, \omega)$

د) $y = [3x] - 3x$

$-1 < [3x] - 3x \leq 0$

$R_f = (-1, 0]$

$$\text{اذا } y = P \sin x + Q \cos x$$

$$-\sqrt{P^2+Q^2} \leq P \sin x + Q \cos x \leq \sqrt{P^2+Q^2}$$

$$[-\sqrt{14}, \sqrt{14}] = R_f$$

$$\rightarrow y = P \sin x - P \cos x \quad R_f = [-\sqrt{2}, \sqrt{2}]$$

$$-\sqrt{14+9} \leq P \sin x - P \cos x \leq \sqrt{14+9}$$

$$\text{ج) } y = [P \sin x + Q \cos x]$$

$$-\sqrt{P^2+Q^2} \leq P \sin x + Q \cos x \leq \sqrt{P^2+Q^2}$$

$$-\sqrt{14} \leq y \leq \sqrt{14}$$

$$[y] \rightarrow \{-\omega, -\frac{P}{Q}, -\frac{P}{Q}, -1\}$$

$$R_f \leftarrow \{0, 1, P, P, P, \omega\}$$

$$\text{د) } \sqrt{\cos x - P \sin x} \quad R_f = [0, \sqrt{2}]$$

$$-\sqrt{1+P} \leq \cos x - P \sin x \leq \sqrt{2} \rightarrow [0, \sqrt{2}]$$

$$\text{اذا } y = \sin^2 x + P \sin x + P$$

$$1 \sqrt{\frac{1}{4}} \quad -1 \quad -\frac{P}{2} x$$

$$R_f = [0, 4]$$

$$\text{ج) } y = P \sin^2 x + P \sin x + P$$

$$-\frac{P}{2} \downarrow \quad \downarrow \quad \downarrow -1$$

$$R_f = [\frac{P}{4}, P]$$

$$\text{اذا } \sin^2 x + P \cos^2 x$$

$$(1 - \sin^2 x)$$

$$\sin^2 x + P - P \sin^2 x = P - P \sin^2 x$$

$$0 \sqrt{\frac{1}{4}} \quad 1 \quad -\frac{1}{2} \omega$$

$$R_f = [1, P]$$

$$\text{ج) } y = \frac{P \cot x}{1 + \cot^2 x}$$

$$1 + \cot^2 x \neq 0$$

$$\cot x \rightarrow 1$$

$$R_f = [-1, 1]$$

$$\text{اذا } y = \sin^4 x + \cos^4 x$$

$$P \frac{1-P}{2} \leq y \leq 1$$

$$\frac{1}{P} \leq y \leq 1$$

$$R_f = [\frac{1}{P}, 1]$$

$$\text{ج) } y = [\sin^4 x + \cos^4 x]$$

$$P \frac{1-P}{2} \leq \sin^4 x + \cos^4 x \leq 1$$

$$\frac{1}{P} \leq \sin^4 x + \cos^4 x \leq 1$$

$$R_f = \left\{ \frac{1}{2} \right\}$$

$$\text{اذا } y = \frac{Pn^2 + \sqrt{n-P}}{n+P}$$

$$\frac{(n+P)(Pn-1)}{n+P} = Pn-1$$

$$n \neq P \rightarrow y \neq -1$$

$$R_f = (R - \{-1\})$$

$$\text{ج) } y = \frac{n^2-1}{n-1} = \frac{(n-1)(n^2+n+1)}{n-1}$$

$$\rightarrow n^2+n+1 \rightarrow \left[\frac{P}{P}, +\infty \right)$$

$$n \neq 1 \rightarrow y \neq P$$

$$R_f = \left[\frac{P}{P}, +\infty \right)$$

