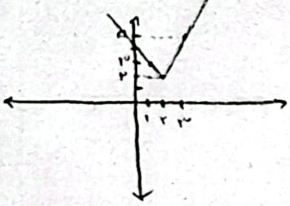
نام و نام خانوادگی ....هلیا عطائی.... پاسخنامه تشریحی تکلیف شماره ...۷... کلاس ...یازدهم دختر...

۱

الف) $x + |2x-4|$

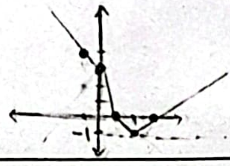
$$\begin{array}{c|c} & 2 \\ x - 2x + 4 = -x+4 & x + 2x - 4 = 3x-4 \end{array}$$



$R_f : [2, +\infty)$

ب) $y = |x-2| + |x-1| - |x|$

$$\begin{array}{c|c|c|c} & 0 & 1 & 2 \\ -x+2-x+1+x = \boxed{-x+3} & -x+2-x+1-x = \boxed{-3x+3} & -x+2+x-1-x = \boxed{-x+1} & x-2+x-1-x = \boxed{x-3} \end{array}$$



$R_f : [-1, +\infty)$

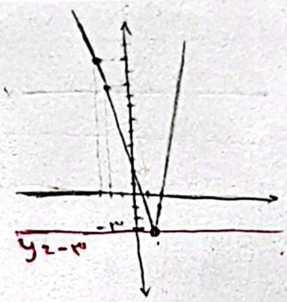
۲

$|\underbrace{3x-3}_{x=1}| - |\underbrace{x+2}_{x=-2}| = k$

$$\begin{array}{c|c|c} & -2 & 1 \\ -3x+3+x+2 = \boxed{-2x+5} & -3x+3-x-2 = \boxed{-4x+1} & 3x-3-x-2 = \boxed{2x-5} \end{array}$$

$$\begin{cases} y = |3x-3| - |x+2| \\ y = k \end{cases} \xrightarrow{\text{از روی شکل می‌توان دید}} (k = -3)$$

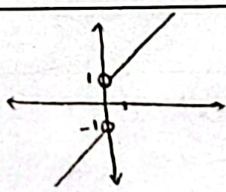
فقط خط $y=-3$ و شکل را در یک نقطه قطع می‌کند



۳

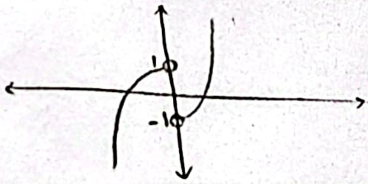
الف) $y = x + \frac{x}{|x|}$

$$\begin{array}{c|c} & 0 \\ x + \frac{x}{-x} - 1 = \boxed{x-1} & x + \frac{x}{x}\ 1 = \boxed{x+1} \end{array}$$



ب) $y = x|x| - \frac{x}{|x|}$

$$\begin{array}{c|c} & 0 \\ -x^2 - \frac{x}{-x}\ -1 = \boxed{-x^2+1} & x^2 - \frac{x}{x}\ 1 = \boxed{x^2-1} \end{array}$$



۴

الف) $y = [x] - 2x$

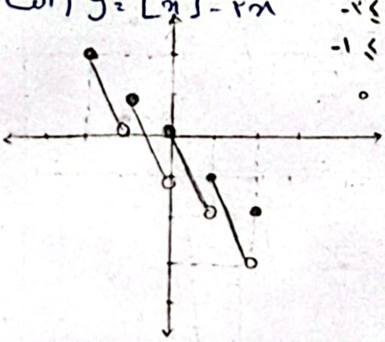
$-2 \le x < -1 \xrightarrow{y=} -2-2x$

$-1 \le x < 0 \xrightarrow{y=} -1-2x$

$0 \le x < 1 \xrightarrow{y=} 0 - 2x = -2x$

$1 \le x < 2 \xrightarrow{y=} 1-2x$

$x = 2 \to y = 2-4 = -2$



ب) $[x]|x|$

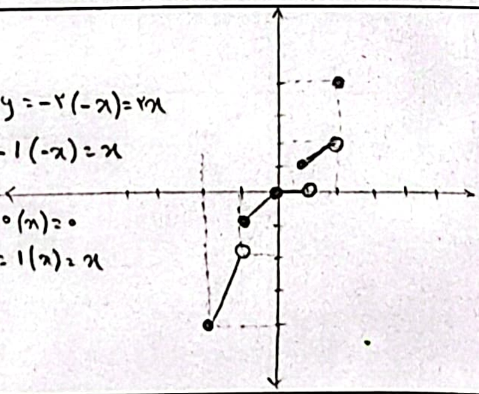
$-2 \le x < -1 \to y = -2(-x) = 2x$

$-1 \le x < 0 \to y = -1(-x) = x$

$0 \le x < 1 \to y = 0(x) = 0$

$1 \le x < 2 \to y = 1(x) = x$

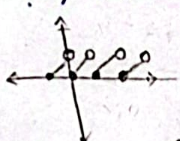
$x = 2 \to y = 4$



۵

الف) $y = 3x - [3x] = [0, 1)$

ب) $4x - 4[x] = 4(x - [x])$



$\to 0 \le y < 1 \xrightarrow{\times 4} 0 \le y < 4$

$R_f : [0, 4)$

ج) $y = 5\frac{x}{5} - 5\left[\frac{x}{5}\right] = 5\overbrace{\left(\frac{x}{5} - \left[\frac{x}{5}\right]\right)}^{(t - [t])}$

$0 \le \frac{x}{5} - \left[\frac{x}{5}\right] < 1 \xrightarrow{\times 5} 0 \le y < 5$

$R_f : [0, 5)$

د) $[2x] - 2x$

$R_f : (-1, 0]$

$$\star: -\sqrt{a^2+b^2} \le a\sin x + b\cos x \le +\sqrt{a^2+b^2}$$

الف) $y = 3\sin x + 2\cos x$

$a^2+b^2 = 9+4=13 \xrightarrow{\star} -\sqrt{13} \le 3\sin x + 2\cos x \le +\sqrt{13}$

$\Rightarrow R_f: [-\sqrt{13}, \sqrt{13}]$

ب) $y = 4\sin x - 3\cos x$

$a^2+b^2 = 16+9=25 \xrightarrow{\star} -\sqrt{25} \le 4\sin x - 3\cos x \le \sqrt{25}$

$\Rightarrow -5 \le 4\sin x - 3\cos x \le 5$

$\Rightarrow R_f: [-5, 5]$ 6

ج) $y = [\underbrace{2\sin x + 5\cos x}_{\min}]$ $\quad \overset{5}{\sqrt{25}} < \sqrt{29} < \overset{6}{\sqrt{36}}$

$a^2+b^2 = 4+25=29 \xrightarrow{\star} -\sqrt{29} \le 2\sin x + 5\cos x \le \sqrt{29} \xrightarrow{[\;]} R_f = \{y \mid y \in \mathbb{Z}, -6 \le y \le 5\}$

$\Rightarrow R_f: \{-6, \pm5, \pm4, \pm3, \pm2, \pm1, 0\}$

د) $y = \sqrt{\cos x - 2\sin x}$

$a^2+b^2 = 1+4=5 \xrightarrow{\star} -\sqrt5 \le \cos x - 2\sin x \le \sqrt5$

$R_f: \sqrt{[-\sqrt5, \sqrt5]} = [0, \sqrt[4]{5}]$

الف) $y = \sin^2 x + 3\sin x + 2$

$\begin{cases} \xrightarrow{\min} x=-1 \Rightarrow y = 1-3+2 = 0 \\ \xrightarrow{\max} x=1 \Rightarrow y = 1+3+2 = 6 \\ \xrightarrow{\frac{-b}{2a}} x = \frac{-3}{2} \Rightarrow \sin x \neq -\frac{3}{2} \quad (-1 \le \sin x \le 1) \end{cases}$

$R_f: [0, 6]$

ب) $y = 2\sin^2 x + 3\sin x + 2$

$\begin{cases} \xrightarrow{\min} x=-1 \Rightarrow 2-3+2 = 1 \\ \xrightarrow{\max} x=1 \Rightarrow 2+3+2 = 7 \\ \frac{-b}{2a} = \frac{-3}{4} \Rightarrow 2 \times \frac{9}{16} + 3 \times \frac{-3}{4} + 2 \end{cases}$

$= \frac{9}{8} - \frac{18}{8} + \frac{16}{8} = \frac{7}{8}$

$R_f: [\frac{7}{8}, 7]$ 7

الف) $y = \underbrace{\sin^2 x + 4\cos^2 x - 1}_{-\cos^2 x} + 1 = 4\cos^2 x - \cos^2 x + 1 \xrightarrow[(\sin^2 x + \cos^2 x = 1)]{} y = 3\cos^2 x + 1$

$\begin{cases} \xrightarrow{\max} x=1 \rightarrow y = 4 \\ \xrightarrow{\min} x=-1 \rightarrow y = 4 \\ \xrightarrow{\frac{-b}{2a}} x=0 \rightarrow y = 1 \end{cases}$ $\quad R_f = [1, 4]$

ب) $y = \frac{2\cot x}{1+\cot^2 x} \xrightarrow[\cot x = \frac{\cos x}{\sin x}]{1+\cot^2 x = \frac{1}{\sin^2 x}} y = \frac{2\cos x}{\cancel{\sin x}} \times \sin^{\cancel{2}} x \rightarrow y = 2\cos x \sin x = \sin 2x \rightarrow -1 \le \sin 2x \le 1$

$R_f: [-1, 1]$ 8

الف) $y = \sin^6 x + \cos^6 x \xrightarrow[2^{1-k} = 2^{-2}]{\star\ 2k=6,\ k=3} \frac{1}{4} \le \sin^6 x + \cos^6 x \le 1 \longrightarrow R_f: [\frac{1}{4}, 1]$

طبق رابطه‌ی $2^{1-k} \le \sin^{2k} x + \cos^{2k} x \le 1$: ★ سوالات حل می‌شوند

ب) $y = [\sin^8 x + \cos^8 x] \xrightarrow[2^{1-k} = 2^{-3}]{\star\ 2k=8,\ k=4} \frac{1}{8} \le \sin^8 x + \cos^8 x \le 1$ 9

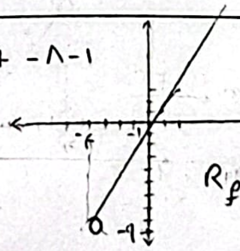
$\xrightarrow{[\;]} [\sin^8 x + \cos^8 x] = \{0, 1\} \rightarrow R_f: \{0, 1\}$

الف) $y = \frac{2x^2+7x-4}{x+4 \neq 0} = \frac{\cancel{(x+4)}(2x-1)}{\cancel{(x+4)} \neq 0} = 2x-1$

$x \neq -4 \rightarrow 2x-1 \neq -8-1$

$y \neq -9$

$2x^2+7x-4 = x^2+7x-8 = (x+\frac{8}{2})(2x-1) = (x+4)(2x-1)$

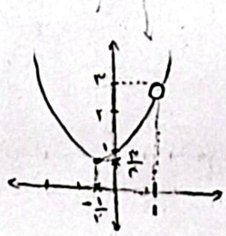


$R_f = \mathbb{R} - \{-9\}$ 10

ب) $y = \frac{x^3-1}{x-1 \neq 0} = \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)} \neq 0} = x^2+x+1$

$a>0 \xrightarrow{} b\min$

$\begin{cases} x_{\min} = \frac{-b}{2a} = \frac{-1}{2} \\ y_{\min} = \frac{1}{4} - \frac{2}{4} + \frac{4}{4} = \frac{3}{4} \end{cases}$



$\xrightarrow{x \neq 1} x^2+x+1 \neq 3 \rightarrow$

$R_f: [\frac{3}{4}, +\infty)$