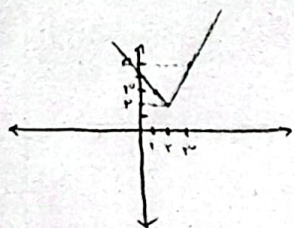


الف) $x + |x-4|$

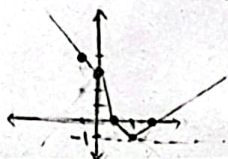


$$\begin{array}{r|l} x & x+|x-4| \\ \hline x & x+4 \\ x-4 & x-4 \end{array}$$

$R_f: [4, +\infty)$

ب) $y = |x-2| + |x-1| - |x|$

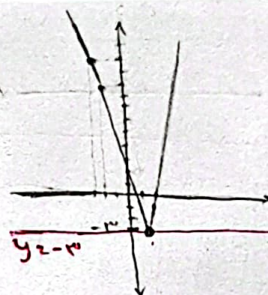
$$\begin{array}{r|l} x & |x-2| + |x-1| - |x| \\ \hline x & -x+2+x-1-x = -x+1 \\ x-1 & -x+2+x-1-1 = -x \\ x-2 & -x+2+x-1-1 = -x \end{array}$$



$R_f: [-1, +\infty)$

$|3x-3| - |x+2| = k$

$$\begin{array}{r|l} x & |3x-3| - |x+2| \\ \hline x & 3x-3-x-2 = 2x-5 \\ x-1 & 3x-3-x-2 = 2x-5 \\ x-3 & 3x-3-x-2 = 2x-5 \end{array}$$



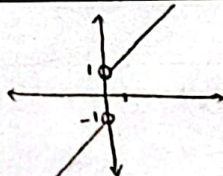
$y = |3x-3| - |x+2|$

از روی شکل می توان گفت
شماره ۳ و شکل ۱ است به نظر می آید

$k = -3$

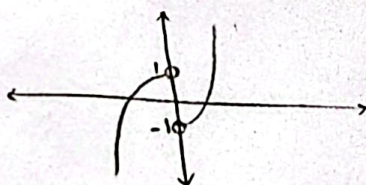
الف) $y = x + \frac{x}{|x|}$

$$\begin{array}{r|l} x & x + \frac{x}{|x|} \\ \hline x & x + \frac{x}{x} = x+1 \\ x-1 & x + \frac{x}{x-1} \end{array}$$



ب) $y = x|x| - \frac{x}{|x|}$

$$\begin{array}{r|l} x & x|x| - \frac{x}{|x|} \\ \hline x & x^2 - \frac{x}{x} = x^2 - 1 \\ x-1 & (x-1)|x-1| - \frac{x-1}{|x-1|} \end{array}$$



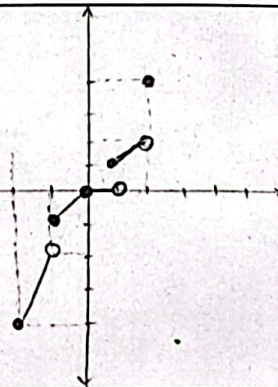
الف) $y = [x] - 2x$



$$\begin{array}{l} -1 \leq x < 0 \rightarrow y = -1 - 2x \\ 0 \leq x < 1 \rightarrow y = 0 - 2x = -2x \\ 1 \leq x < 2 \rightarrow y = 1 - 2x \\ x = 2 \rightarrow y = 2 - 2 = 0 \end{array}$$

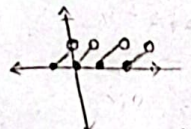
ب) $[x]/|x|$

$$\begin{array}{l} -1 \leq x < 0 \rightarrow y = -2(-x) = 2x \\ -1 \leq x < 0 \rightarrow y = -1(-x) = x \\ 0 \leq x < 1 \rightarrow y = 0(x) = 0 \\ 1 \leq x < 2 \rightarrow y = 1(x) = x \\ x = 2 \rightarrow y = 2 \end{array}$$



الف) $y = 3x - [3x] = \{0, 1\}$

ب) $f(x) - f([x]) = f(x - [x])$



$R_f: [0, 1)$

ج) $y = \frac{x}{a} - \left[\frac{x}{a} \right] = \left\{ \frac{x}{a} - \left[\frac{x}{a} \right] \right\}$

$R_f: [0, 1)$

$0 \leq \frac{x}{a} - \left[\frac{x}{a} \right] < 1 \xrightarrow{\times a} 0 \leq y < a$

$R_f: (-1, 0]$

$$x: \sqrt{a^2+b^2} \leq a \sin x + b \cos x \leq \sqrt{a^2+b^2}$$

الف) $y = r \sin x + r \cos x$

$a+b = 9+9 = 18$
 $\frac{x}{\sqrt{18}} \leq r \sin x + r \cos x \leq \sqrt{18}$

$\Rightarrow R_f: [\sqrt{18}, \sqrt{18}]$

ب) $y = r \sin x - r \cos x$

$a+b = 14+9 = 23$
 $\frac{x}{\sqrt{23}} \leq r \sin x - r \cos x \leq \sqrt{23}$

$\Rightarrow -\sqrt{23} \leq r \sin x - r \cos x \leq \sqrt{23}$

$\Rightarrow R_f: [-\sqrt{23}, \sqrt{23}]$

ج) $y = [r \sin x + 2 \cos x]$
 $\frac{x}{\sqrt{5}} \leq r \sin x + 2 \cos x \leq \sqrt{5}$

$b' = r + 2 = 29$
 $R_f: \{-4, -2, 0, 2, 4\}$

د) $y = \sqrt{\cos x - r \sin x}$

$\frac{x}{\sqrt{2}} \leq \cos x - r \sin x \leq \sqrt{2}$

$R_f: [-\sqrt{2}, \sqrt{2}] = [0, \sqrt{2}]$

الف) $y = \sin^2 x + r \sin x + r$

$\min \Rightarrow x = -1 \Rightarrow y = 1 - r + r = 0$

$\max \Rightarrow x = 1 \Rightarrow y = 1 + r + r = 4$

$\frac{-b}{2a} = \frac{-r}{2} \Rightarrow \sin x = -\frac{r}{2} \quad (-1 \leq \sin x \leq 1)$

$R_f: [0, 4]$

ب) $y = r \sin^2 x + r \sin x + r$

$\min \Rightarrow x = -1 \Rightarrow r - r + r = 1$

$\max \Rightarrow x = 1 \Rightarrow r + r + r = 3$

$\frac{-b}{2a} = \frac{-r}{2} \Rightarrow x = \frac{9}{18} + r \times \frac{-r}{2} + r$
 $= \frac{9}{18} - \frac{18}{18} + \frac{18}{18} = \frac{9}{18}$

$R_f: [\frac{9}{18}, 3]$

الف) $y = \sin^2 x + r \cos^2 x - 1 + 1 = r \cos^2 x - \cos^2 x + 1$

$y = r \cos^2 x + 1$
 $(\sin^2 x + \cos^2 x = 1)$

$\max \Rightarrow x = 1 \Rightarrow y = r$
 $\min \Rightarrow x = -1 \Rightarrow y = 1$
 $R_f: [1, r]$

ب) $y = \frac{\cot x}{1 + \cot^2 x} \cdot \frac{1 + \cot^2 x}{\cot x} = \frac{r \cos x}{\sin x} \cdot \frac{\sin x}{\sin x} \rightarrow y = r \cos x \sin x = \sin 2x$

$R_f: [-1, 1]$

الف) $y = \sin^4 x + \cos^4 x$
 $\frac{x}{\sqrt{2}} \leq \sin^4 x + \cos^4 x \leq \sqrt{2}$

$\frac{1}{\sqrt{2}} \leq \sin^4 x + \cos^4 x \leq 1 \rightarrow R_f: [\frac{1}{\sqrt{2}}, 1]$

ب) $y = [\sin^2 x + \cos^2 x]$
 $\frac{x}{\sqrt{2}} \leq \sin^2 x + \cos^2 x \leq \sqrt{2}$

$\frac{1}{\sqrt{2}} \leq \sin^2 x + \cos^2 x \leq 1$

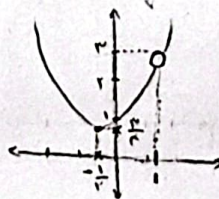
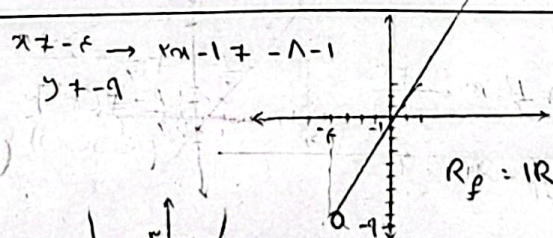
$[1] \rightarrow [\sin^2 x + \cos^2 x] = \{0, 1\} \rightarrow R_f: \{0, 1\}$

الف) $y = \frac{rx^2 + vx - f}{x + f} = \frac{(x+f)(x-1)}{(x+f)} = x-1$

$rx^2 + vx - f = x^2 + vx - 1 = (x+f)(x-1) = (x+f)(rx-1)$

ب) $y = \frac{x^2 - 1}{x - 1} = \frac{(x-1)(x+1)}{(x-1)} = x+1$

$\begin{cases} x_{\min} = \frac{-b}{2a} = \frac{-1}{2} \\ y_{\min} = \frac{1}{4} - \frac{1}{4} + \frac{1}{4} = \frac{1}{4} \end{cases}$



$x \neq 1 \rightarrow x^2 + x + 1 \neq 1 \rightarrow R_f: [\frac{1}{4}, +\infty)$