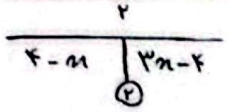
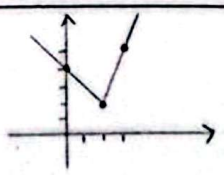
نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

۱

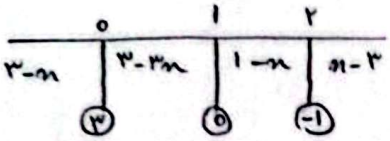
الف) $y = x + |2x-4|$

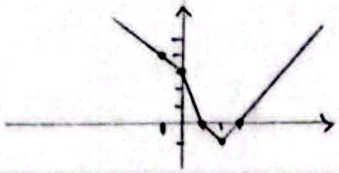




$\Rightarrow R_f = [2, +\infty)$

ب) $y = |x-2| + |x-1| - |x|$





$\Rightarrow R_f = [-1, +\infty)$

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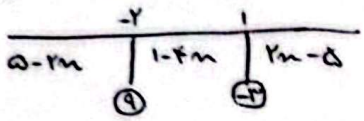
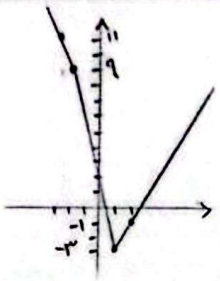
$y = |2x-2| - |x+2|$





$\Rightarrow$ $y=k$ با خط فقط یک برخورد

$k = -۳$

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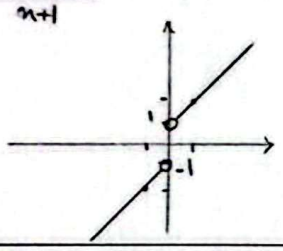
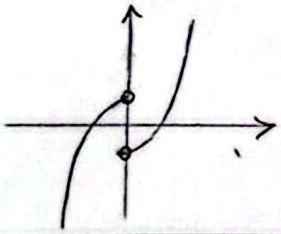
الف) $y = x + \frac{x}{|x|}$

$x-1$	$\nexists$	$x+1$
	۰	



ب) $y = x|x| - \frac{x}{|x|}$

$1-x^2$	$\nexists$	x^2-1
	۰	



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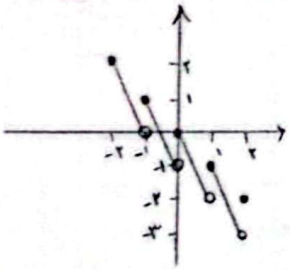
الف) $y = [x] - 2x$



$-2 \le x < -1 \Rightarrow y = -2x-2$

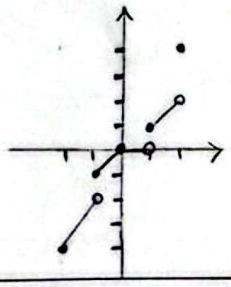
$-1 \le x < 0 \Rightarrow y = -2x-1$

$0 \le x < 1 \Rightarrow y = -2x$

$1 \le x < 2 \Rightarrow y = -2x+1$

$x = 2 \Rightarrow y = -2$

ب) $y = [x]|x|$



$-2 \le x < -1 \Rightarrow y = 2x$

$-1 \le x < 0 \Rightarrow y = x$

$0 \le x < 1 \Rightarrow y = 0$

$1 \le x < 2 \Rightarrow y = x$

$x = 2 \Rightarrow y = 4$

۵

الف) $y = 2x - [2x] \Rightarrow R_f = [0,1)$

ب) $y = 2x - 2[x] = 2\underbrace{(x-[x])}_{[0,1)} \Rightarrow R_f = [0,2)$

ج) $y = x - 5\left[\frac{x}{5}\right] = 5\underbrace{\left(\frac{x}{5} - \left[\frac{x}{5}\right]\right)}_{[0,1)} \Rightarrow R_f = [0,5)$

» $y = [2x] - 2x \rightarrow 0 \le 2x - [2x] < 1 \xrightarrow{\times -1} -1 < [2x] - 2x \le 0 \rightarrow R_f = (-1,0]$

الف) $y = 3\sin x + 2\cos x \rightarrow \sqrt{a^2+b^2} = \sqrt{9+4} = \sqrt{13} \rightarrow R_f = [-\sqrt{13}, \sqrt{13}]$

ب) $y = 4\sin x - 3\cos x \rightarrow \sqrt{a^2+b^2} = \sqrt{16+9} = 5 \rightarrow R_f = [-5, 5]$

ج) $y = [2\sin x + 5\cos x] \rightarrow \sqrt{a^2+b^2} = \sqrt{4+25} = \sqrt{29} \rightarrow [-\sqrt{29}, \sqrt{29}] \rightarrow R_f = \{-6, \pm 5, \pm 4, \pm 3, \pm 2, \pm 1, 0\}$

د) $y = \sqrt{\cos x - 2\sin x} \rightarrow \sqrt{a^2+b^2} = \sqrt{1+4} = \sqrt{5} \rightarrow \sqrt{[-\sqrt{5}, \sqrt{5}]} \Rightarrow R_f = [0, 2\sqrt{5}]$

۶

الف) $y = \sin^2 x + 2\sin x + 2$

$\sin x$: $1 \rightarrow 1+2+2 = 5$, $-1 \rightarrow 1-2+2 = 0$

$\frac{-b}{2a} = \frac{-2}{2} \rightarrow \frac{9}{4} - \frac{9}{2} + 2 = \frac{-1}{4}$ بیشتر

$\Rightarrow R_f = [0, 5]$

ب) $y = 2\sin^2 x + 2\sin x + 2$

$\sin x$: $1 \rightarrow 2+2+2 = 7$, $-1 \rightarrow 2-2+2 = 1$

$\frac{-b}{2a} = \frac{-2}{4} \rightarrow \frac{9}{8} - \frac{9}{4} + 2 = \frac{7}{8}$

$\Rightarrow R_f = [\frac{7}{8}, 7]$

۷

الف) $y = \sin^2 x + \underbrace{4\cos^2 x}_{\cos^2 x + 3\cos^2 x} = \underbrace{(\sin^2 x + \cos^2 x)}_{1} + \underbrace{3\cos^2 x}_{[0,3]} \rightarrow R_f = [1, 4]$

ب) $y = \frac{2\cot x}{1+\cot^2 x} = \frac{2\cos x}{\sin x} \times \sin^2 x = 2\sin x \cos x = \sin 2x$

$\Rightarrow R_f = [-1, 1]$

۸

الف) $y = \sin^4 x + \cos^4 x \rightarrow \underbrace{2^{1-2}}_{\frac{1}{2}} \left|\sin^{2\times 2} x + \cos^{2\times 2} x\right| \leq 1 \Rightarrow R_f = [\frac{1}{2}, 1]$

ب) $y = [\sin^8 x + \cos^8 x] \Rightarrow [[\frac{1}{8}, 1]] \Rightarrow R_f = \{0, 1\}$

$\underbrace{2^{1-4}}_{\frac{1}{8}} \left|\sin^{2\times 4} x + \cos^{2\times 4} x\right| \leq 1$

۹

الف) $y = \frac{2x^2 + 7x - 4}{x+4} = \frac{(x+4)(2x-1)}{(x+4)} = 2x - 1 \rightarrow R = \mathbb{R}$, $x \neq -4 \rightarrow R = -9$ $\Big\} R_f = \mathbb{R} - \{-9\}$

ب) $y = \frac{x^3 - 1}{x - 1} = \frac{(x-1)(x^2+x+1)}{(x-1)} = x^2 + x + 1$

$\min \left| \begin{matrix} -\frac{1}{2} \\ \frac{3}{4} \end{matrix} \right.$ $\Rightarrow R_f = [\frac{3}{4}, +\infty)$

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