باران کاظمی   یازدهم   تکلیف شماره‌ی ۷

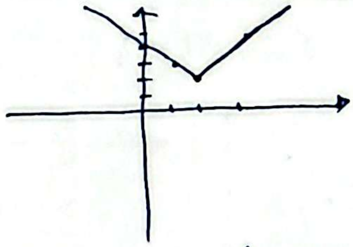
-۱

الف) $y = x + |2x - 4|$

	2	
$x - 2x + 4$		$x + 2x - 4$
$-x + 4$		$3x - 4$

1	2	3
3	2	5

$R_y = [2, +\infty)$

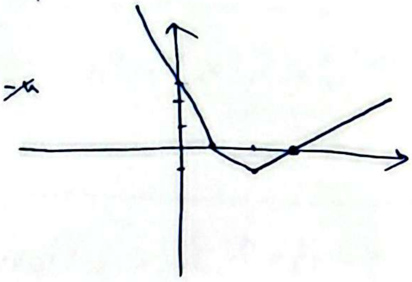


ب) $y = |x - 2| + |x - 1| - |x|$

	0		1		2	
$-x+2-x+1+x$		$-x+2-x+1-x$		$-x+2+x-1-x$		$x-2+x-1-x$
$-x+3$		$-3x+3$		$-x+1$		$x-3$

0	1	2	3
3	0	-1	0

$R_y = [-1, +\infty)$



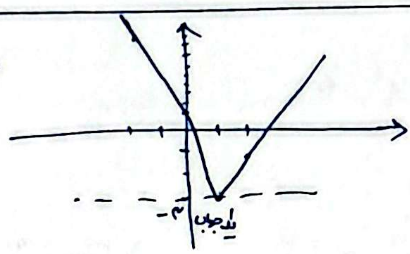
-۲

$|3x - 3| - |x + 2| = k$

	-2		1	
$-3x+3+x+2$		$-3x+3-x-2$		$3x-3-x-2$
$-2x+5$		$-4x+1$		$2x-5$

-3	-2	1	2
11	9	-3	-1

$k = -3$

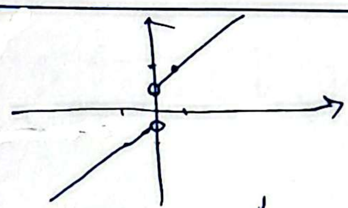


-۳

الف) $y = x + \dfrac{x}{|x|}$

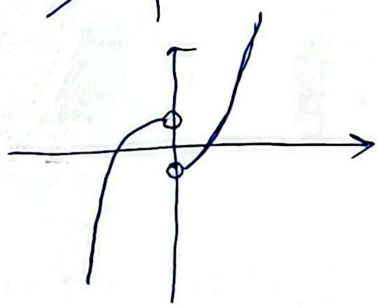
	0	
$x - 1$		$x + 1$

-1	1
-2	2



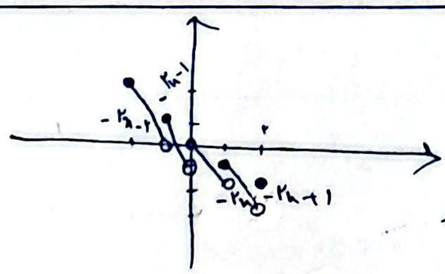
ب) $y = x|x| - \dfrac{x}{|x|}$

	0	
$-x^2 + 1$		$x^2 - 1$



-۴ $[-2, 2]$

الف) $y = [x] - 2x$



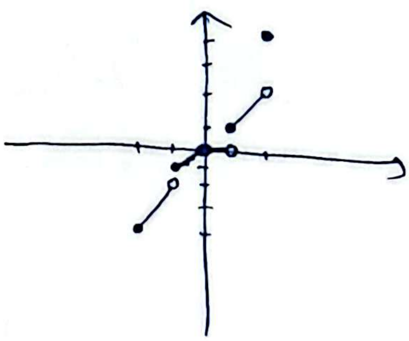
ب) $y = [x]|x|$

$[-2, -1) \to [x] = -2 \to 2x$

$[-1, 0) \to [x] = -1 \to x$

$[0, 1) \to [x] = 0 \to 0$

$[1, 2) \to x$



-۵

الف) $y = 3x - [3x]$

$0 \le x - [x] < 1 \Rightarrow 0 \le 3x - [3x] < 1$ $\boxed{R_y = [0,1)}$

ب) $y = 4x - 4[x]$ $0 \le x - [x] < 1 \Rightarrow 0 \le 4x - 4[x] < 4$ $\boxed{R_y = [0,4)}$

ج) $y = x - 5[\frac{x}{5}] \Rightarrow 5(\frac{x}{5} - [\frac{x}{5}])$ $0 \le \frac{x}{5} - [\frac{x}{5}] < 1 \Rightarrow 0 \le 5(\frac{x}{5} - [\frac{x}{5}]) < 5$

$\boxed{R_y = [0,5)}$

د) $y = [2x] - 2x$ $-1 < [2x] - 2x \le 0$ $\boxed{R_y = (-1,0]}$

-۶

الف) $y = 3\sin x + 2\cos x$ $-\sqrt{9+4} \le 3\sin x + 2\cos x \le \sqrt{9+4}$

$-\sqrt{13} \le 3\sin x + 2\cos x \le \sqrt{13} \Rightarrow R_y = [-\sqrt{13}, \sqrt{13}]$

ب) $y = 4\sin x - 3\cos x$ $-\sqrt{25} \le 4\sin x - 3\cos x \le \sqrt{16+9}$

$\Rightarrow -5 \le 4\sin x - 3\cos x \le 5 \Rightarrow R_y = [-5,5]$

ج) $y = [2\sin x + 5\cos x]$ $-\sqrt{29} \le 2\sin x + 5\cos x \le \sqrt{29}$ $\Rightarrow R_y = \{-6,-5,-4,-3,-2,-1,0,1,2,3,4,5\}$

$-6 \le [2\sin x + 5\cos x] \le 5$

د) $y = \sqrt{\cos x - 2\sin x}$ $-\sqrt{5} \le \cos x - 2\sin x \le \sqrt{5} \Rightarrow 0 \le \sqrt{\cos x - 2\sin x} \le \sqrt[4]{5} \Rightarrow R_y = [0, \sqrt[4]{5}]$

-۷

الف) $y = \sin^2 x + 3\sin x + 2$

- $-1 \Rightarrow 1 - 3 + 2 = 0$
- $1 \Rightarrow 1 + 3 + 2 = 6$
- $-\frac{b}{2a} = -\frac{3}{2}$ $-1 \le \sin x \Rightarrow \underset{\sin x}{-\frac{3}{2}}$ غ ق ق

$R_y = [0,6]$

ب) $y = 2\sin^2 x + 3\sin x + 2$

- $-1 \to 2 - 3 + 2 = 1$
- $1 \to 2 + 3 + 2 = 7$
- $-\frac{b}{2a} = -\frac{3}{4} \Rightarrow 2(\frac{9}{16}) + 3(-\frac{3}{4}) + 2 = \frac{9 - 18 + 16}{8} = \frac{7}{8}$

$R_y = [\frac{7}{8}, 7]$

-۸

الف) $y = \sin^2 x + 4\cos^2 x = 1 - \cos^2 x + 4\cos^2 x = 3\underbrace{\cos^2 x}_{[0,1]} + 1$

$0 \le \cos^2 x \le 1 \Rightarrow 0 \le 3\cos^2 x \le 3 \Rightarrow 1 \le 3\cos^2 x + 1 \le 4$ $R_y = [1,4]$

ب) $y = \frac{2\cot x}{1 + \cot^2 x} = \frac{\frac{2\cos x}{\sin x}}{\frac{1}{\sin^2 x}} = 2\cos x \sin x = \sin 2x$ $R_y = [-1,1]$

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۹-

الف) $y = \sin^4 x + \cos^4 x \Rightarrow 2^{-2} \leq \sin^{2(2)} x + \cos^{2(2)} x \leq 1 \Rightarrow \frac{1}{4} \leq \sin^4 + \cos^4 \leq 1$

$R_y = [\frac{1}{4}, 1]$

ب) $y = [\sin^8 x + \cos^8 x] \Rightarrow 2^{-3} \leq \sin^{2(4)} x + \cos^{2(4)} x \leq 1 \Rightarrow \frac{1}{8} \leq \sin^8 x + \cos^8 x \leq 1$

$0 \leq [\sin^8 x + \cos^8 x] \leq 1 \Rightarrow R_y = [0, 1]$

۱۰-

الف) $y = \frac{2x^2 + 7x - 4}{x + 4} = \frac{\cancel{(x+4)}(2x-1)}{\cancel{(x+4)}}$ $\quad R_y = \mathbb{R} - \{-9\}$

$$\begin{array}{l|l} \cancel{2x^2} + 7x - 4 & x+4 \\ \hline -\cancel{2x^2} \mp 8x & 2x-1 \\ \hline -x-4 & \\ +x+4 & \\ \hline 0\ 0 & \end{array}$$

$= 2x - 1 \to -8 - 1 = -9$

ب) $y = \frac{x^3 - 1}{x - 1} = \frac{\cancel{(x-1)}(x^2 + x + 1)}{\cancel{(x-1)}} = x^2 + x + 1 \Rightarrow \left| \begin{array}{l} -\frac{1}{2} \\ \frac{1}{4} - \frac{1}{2} + 1 = \frac{1 - 2 + 4}{4} = \frac{3}{4} \end{array} \right.$

$\to 1 + 1 + 1 = 3$

$[\frac{3}{4}, +\infty) - \{3\}$

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