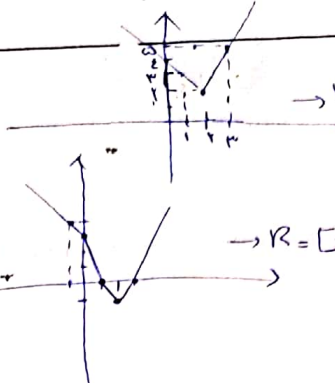


الف) $y = x + |x - 4|$

$$\begin{array}{c|c|c|c|c} x & x+4 & x-4 & |x-4| & y \\ \hline x & x+4 & x-4 & x-4 & x+x-4=2x-4 \\ \hline 4 & 8 & 0 & 0 & 4 \\ \hline x & x+4 & x-4 & 4-x & x+x-4+4-x=4 \\ \hline \end{array}$$

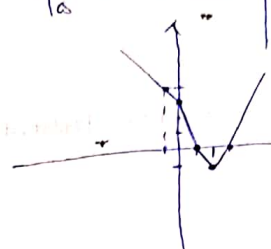
$\rightarrow R = [2, +\infty)$



ب) $y = |x - 2| + |x - 1| - |x - 2|$

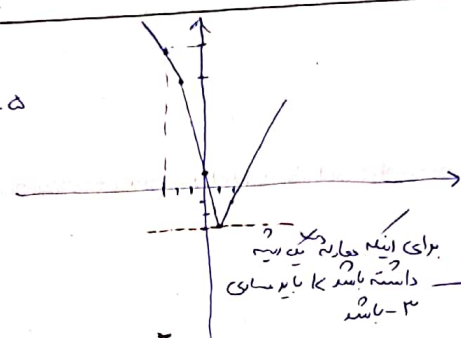
$$\begin{array}{c|c|c|c|c} x & x-2 & x-1 & |x-2| & y \\ \hline x & x-2 & x-1 & x-2 & x+x-2-x-2=-2 \\ \hline 2 & 0 & 1 & 0 & -1 \\ \hline x & x-2 & x-1 & 2-x & x+x-2-2+x=x-2 \\ \hline \end{array}$$

$\rightarrow R = [-1, +\infty)$



$|3x - 2| - |x + 2| = 1$

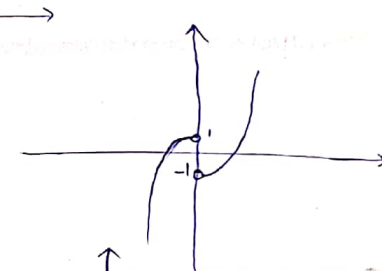
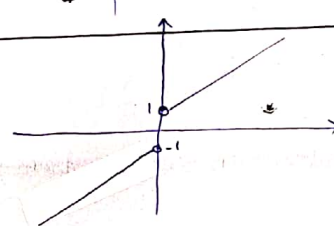
$$\begin{array}{c|c|c|c|c} x & 3x-2 & x+2 & |3x-2| & y \\ \hline x & 3x-2 & x+2 & 3x-2 & 3x-2-x-2=2x-4 \\ \hline 2 & 4 & 4 & 4 & 0 \\ \hline x & 3x-2 & x+2 & 2-3x & 3x-2-x-2+2-3x=-2 \\ \hline \end{array}$$



الف) $y = x + \frac{x}{|x|}$

ب) $y = x|x| - \frac{x}{|x|}$

$$\begin{array}{c|c|c|c|c} x & x & \frac{x}{|x|} & x|x| & y \\ \hline x & x & \frac{x}{|x|} & x|x| & x|x| - \frac{x}{|x|} \\ \hline \end{array}$$

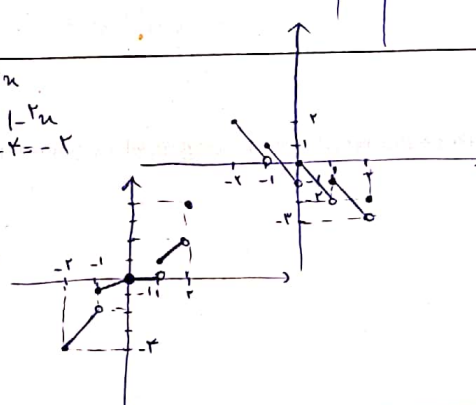


الف) $y = [x] - 2x$

$[2, 3)$ در بازه $\rightarrow -2 - 2x$ در بازه $[1, 2)$ $\rightarrow -1 - 2x$ در بازه $[0, 1)$ $\rightarrow 0 - 2x$ در بازه $[-1, 0)$ $\rightarrow -1 - 2x$ در بازه $[-2, -1)$ $\rightarrow -2 - 2x$

ب) $y = [x]|x|$

$[2, 3)$ در بازه $\rightarrow (2)(x) = 2x$ در بازه $[1, 2)$ $\rightarrow (1)(x) = x$ در بازه $[0, 1)$ $\rightarrow (0)(x) = 0$ در بازه $[-1, 0)$ $\rightarrow (-1)(-x) = x$ در بازه $[-2, -1)$ $\rightarrow (-2)(-x) = 2x$



الف) $y = 3x - [3x] \rightarrow R = [0, 1)$

ب) $y = 3x - 4[3x] \rightarrow 0 \leq 3x - 4[3x] < 4 \rightarrow R = [0, 4)$

ج) $y = x - \omega \left[\frac{x}{\omega} \right] \rightarrow R = [0, \omega)$

د) $y = [3x] - 3x \rightarrow -1 \leq [3x] - 3x < 0 \rightarrow R = (-1, 0]$

$$a) y = r \sin n + r \cos n \quad -\sqrt{r^2 + r^2} \leq r \sin n + r \cos n \leq \sqrt{r^2 + r^2} \rightarrow R = [-\sqrt{2}r, \sqrt{2}r]$$

$$b) y = r \sin n - r \cos n \quad -\sqrt{r^2 + (-r)^2} \leq r \sin n - r \cos n \leq \sqrt{r^2 + (-r)^2} \rightarrow R = [-\sqrt{2}r, \sqrt{2}r]$$

$$c) y = [r \sin n + 2 \cos n] \quad \frac{-\sqrt{r^2 + 4}}{\sqrt{r^2 + 4}} \leq r \sin n + 2 \cos n \leq \frac{\sqrt{r^2 + 4}}{\sqrt{r^2 + 4}} \rightarrow R = \{-\sqrt{r^2 + 4}, \sqrt{r^2 + 4}\}$$

$$d) y = \sqrt{\cos n - r \sin n} \quad -\sqrt{1 + (-r)^2} \leq \cos n - r \sin n \leq \sqrt{1 + (-r)^2} \rightarrow R = [-\sqrt{1 + r^2}, \sqrt{1 + r^2}]$$

$$a) y = \sin^2 n + r \cos^2 n \rightarrow y = \sin^2 n + r - r \sin^2 n = r + \sin^2 n (1 - r) \quad \begin{matrix} \sin^2 n = 0 \\ \sin^2 n = 1 \end{matrix} \rightarrow R = [r, 1]$$

$$b) y = \frac{r \cos^2 n}{1 + \cos^2 n} = \frac{r \cos^2 n}{\frac{1}{\sin^2 n}} = r \cos^2 n \sin^2 n \rightarrow R = [-1, 1]$$

Observe

$$a) y = \sin^2 n + r \sin n + r \quad \begin{matrix} \max = 1 \\ \sin n = 1 \\ \sin n = -1 \end{matrix} \quad \begin{matrix} 1 + r + r = 1 + 2r \\ 1 - r - r = 1 - 2r \end{matrix} \rightarrow R = [1 - 2r, 1 + 2r]$$

Observe

$$b) y = r \sin^2 n + r \sin n + r \quad \begin{matrix} \max = 1 \\ \sin n = 1 \\ \sin n = -1 \end{matrix} \quad \begin{matrix} r + r + r = 3r \\ r - r - r = -r \end{matrix} \rightarrow R = [-r, 3r]$$

$$a) y = \sin^2 n + \cos^2 n \quad \frac{1}{r} \leq \sin^2 n + \cos^2 n \leq 1 \rightarrow R = [\frac{1}{r}, 1]$$

$$b) y = [\sin^2 n + \cos^2 n] \quad \frac{1}{r} \leq \sin^2 n + \cos^2 n \leq 1 \rightarrow R = \{\frac{1}{r}, 1\}$$

$$a) y = \frac{r n^2 + \sqrt{n} - r}{n + r} = \frac{(r n - 1)(n + \sqrt{n})}{(n + r)} = r n - 1$$

$$b) y = \frac{r n^2 - 1}{n - 1} = \frac{(n - 1)(n + 1)}{n - 1} = n + 1$$

$$\begin{matrix} \frac{1}{r} \leq n + 1 \leq \frac{r}{r} \\ \frac{1}{r} \leq n + 1 \leq 1 \end{matrix} \rightarrow R = [\frac{1}{r}, 1]$$

