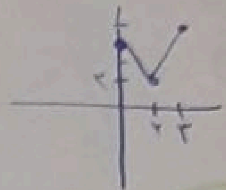
یازدهم (ریاضی)

زهرا آقایی

(1) الف) $y = x + |2x - 4|$

$f(x) = \begin{cases} x + 2x - 4 = 3x - 4 & x > 2 \\ 2 & 2 \\ x - 2x + 4 = -x + 4 & x < 2 \end{cases}$



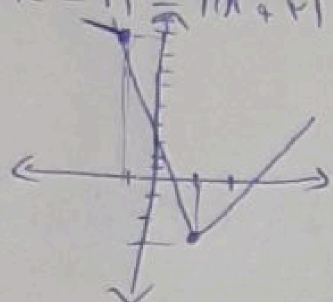
$R(f) = [2, +\infty)$

ب) $y = |x-2| + |x-1| - |x|$

	0	1	2	
$-x+2-x+1+x = -x+3$	$-x+2-x+1-x = -3x+3$	$-x+2+x-1-x = -x+1$	$x-2+x-1-x = x-3$	

$R_f = [-1, +\infty)$

(2) $y = |3x - 3| - |x + 2| = k$



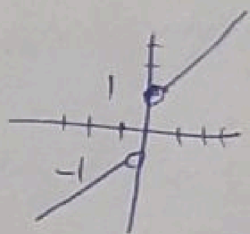
	-2	1	
$-3x+5$	$-4x+1$	$3x-3-x-2 = 2x-5$	

$K = -3$

k باید مقدار کند تا یکبار با نمودار تلاقی داشته باشد

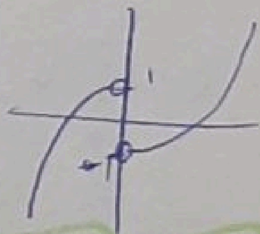
(3) الف) $y = x + \frac{x}{|x|}$

$f(x) \begin{cases} x + 1 & x \geq 0 \\ x - 1 & x < 0 \end{cases}$

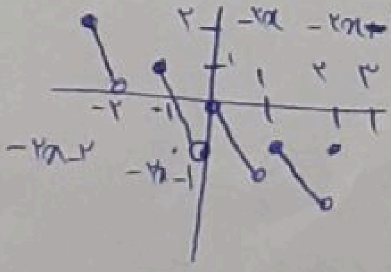


ب) $y = x|x| - \frac{x}{|x|}$

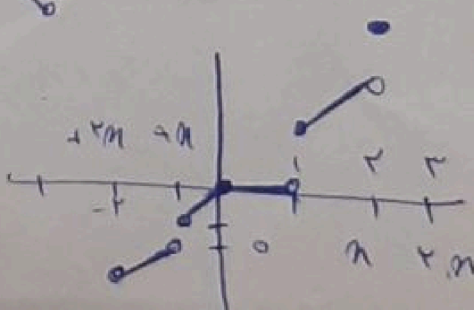
$f(x) \begin{cases} x^2 - 1 & x > 0 \\ -x^2 + 1 & x < 0 \end{cases}$



(4) الف) $y = [x] - 2x$



ب) $y = [x]|x|$



(۵)

الف) $y = 4x - [4x]$ $\quad R(f) = [0, 1)$

ب) $y = 5x - 5[x] \rightarrow 5(x - [x]) \Rightarrow R(f) = [0, 5)$

$0 \leq x - [x] < 1$

ج) $y = x - 5\left[\frac{x}{5}\right] = 5\left(\frac{x}{5} - \left[\frac{x}{5}\right]\right) \rightarrow R(f) = [0, 5)$

$0 \leq \frac{x}{5} - \left[\frac{x}{5}\right] < 1$

د) $y = [2x] - 4x \rightarrow R_f = (-1, 0]$

(۶)

الف) $y = 3\sin x + 4\cos x \rightarrow -\sqrt{9+16} \leq 3\sin x + 4\cos x \leq \sqrt{9+16}$

$R(f) = [-\sqrt{13}, \sqrt{13}]$

ب) $y = 5\sin x - 3\cos x \rightarrow -\sqrt{16+9} \leq 5\sin x + (-3\cos x) \leq \sqrt{16+9}$

$R(f) = [-5, 5]$

ج) $y = [4\sin x + 3\cos x] \rightarrow -\sqrt{25+9} \leq 4\sin x + 3\cos x \leq \sqrt{25+9}$

$R(f) = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

د) $y = \sqrt{\cos x - 2\sin x} \Rightarrow \sqrt{\cos x + (-2\sin x)}$

$-\sqrt{5} < \cos x + (-2\sin x) \leq \sqrt{5} \quad R(f) = [0, \sqrt{5}]$

(۷)

الف) $y = \sin^2 x + 3\sin x + 2$

$\sin x = -1 \quad y = 0 \qquad \sin x = 1 \rightarrow y = 6$

$\sin x = -\frac{3}{4} \rightarrow \alpha \qquad R(f) = [0, 6]$

۷ صفحه (ب)

ب) $y = 2\sin^2 x + 3\sin x + 2$

$\sin x = -1 \rightarrow y = 1$

$\sin x = 1 \rightarrow y = 7$

$\sin x = -\frac{3}{4} \rightarrow y = \frac{9}{8} - \frac{18}{8} + \frac{16}{8} = \frac{7}{8}$

$R(f) = [\frac{7}{8}, 7]$

۸)

الف) $y = \sin^2 x + 4\cos^2 x \rightarrow \sin^2 x + 4 - 4\sin^2 x = -3\sin^2 x + 4$

$4\sin^2 x + 4\cos^2 x = 4$

$4\cos^2 x = 4 - 4\sin^2 x$

$0 \leq \sin^2 \leq 1$

$R(f) = [1, 4]$

ب) $y = \frac{2\cot x}{1+\cot^2 x} = \frac{\frac{2\cos x}{\sin x}}{\frac{1}{\sin^2 x}} = 2\cos x \sin x = \sin(2x)$

$R(f) = [-1, 1]$

۹)

الف) $y = \sin^4 x + \cos^4 x$

$2^{1-k} \leq \sin^{2k} x + \cos^{2k} x \leq 1$

$\frac{1}{4} \leq \sin^4 x + \cos^4 x \leq 1 \rightarrow R(f) = [\frac{1}{4}, 1]$

ب) $y = [\sin^8 x + \cos^8 x]$

$\frac{1}{8}$ ~~2^{-3}~~ $\leq \sin^8 x + \cos^8 x \leq 1$

براکت

$R(f) = \{0, 1\}$

۱۰)

الف) $y = \frac{2x^2 + 7x - 4}{x + 4}$

$2x^2 + 7x - 4 = y \rightarrow \times 2$

$4x^2 + 14x - 8 = 2y$

$(2x + 8)(2x - 1) = 2y \rightarrow y = (x+4)(2x-1)$

$\frac{(x+4)(2x-1)}{(x+4)} = 2x - 1 \rightarrow \mathbb{R}$

$R(f) = \mathbb{R} - \{-9\}$

ب) $\frac{x^3 - 1}{x - 1} = \frac{(x-1)(x^2 + 1 + x)}{x - 1} = x^2 + 1 + x$

$S \,|\, -\frac{1}{2},\ \frac{3}{4}$

$R(f)\ [\frac{3}{4}, +\infty)$