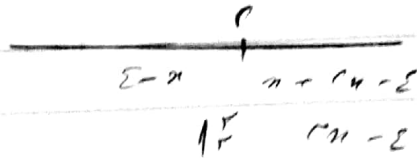
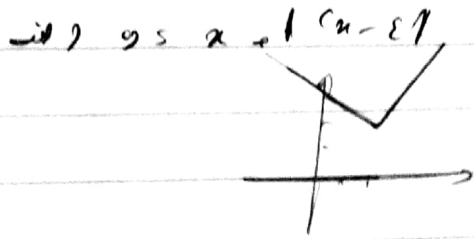


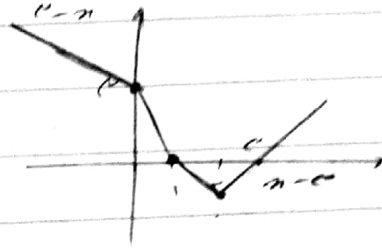
Joined at



$$R_L = [0, \infty)$$

$$\Rightarrow |x - \varepsilon| + |x - 1| - |x| =$$

$$R_L = [-1, \infty)$$

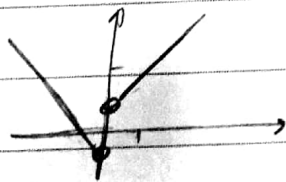


$$(|x - \varepsilon| + |x - 1| - |x|)(k)$$

$$\begin{aligned} (x - \varepsilon)^2 + (x + 1)^2 - 2(x - \varepsilon)(x + 1) &= 9x^2 + 9 - 18x + 6\varepsilon + 2\varepsilon x \\ &= 4x^2 - 16x + 4\varepsilon + 18 = 4x^2 - 16x + 18 = k^2 \\ (x - 2)^2 &= k^2 \end{aligned}$$

$$x = x - 2 \quad \vee \quad x = 2 - x$$

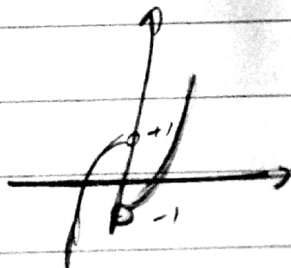
iii) $y \leq x + \frac{x}{|x|}$



$$\frac{0}{x - 1} \leq x + 1$$

$$\Rightarrow y \leq x + \frac{x}{|x|}$$

$$\frac{0}{-x^2 + 1} \leq x - 1$$



$\text{Graph of } y = \lfloor x \rfloor$

1) $y = \lfloor x \rfloor - \pi$

$0 \leq x < 1 \rightarrow y = -\pi$

$1 \leq x < 2 \rightarrow y = 1 - \pi$

$2 \leq x < 3 \rightarrow y = 2 - \pi$

$-1 \leq x < 0 \rightarrow y = -\pi - 1$

$-2 \leq x < -1 \rightarrow y = -\pi - 2$

2) $y = \lfloor x \rfloor$

$0 \leq x < 1 \rightarrow y = 0$

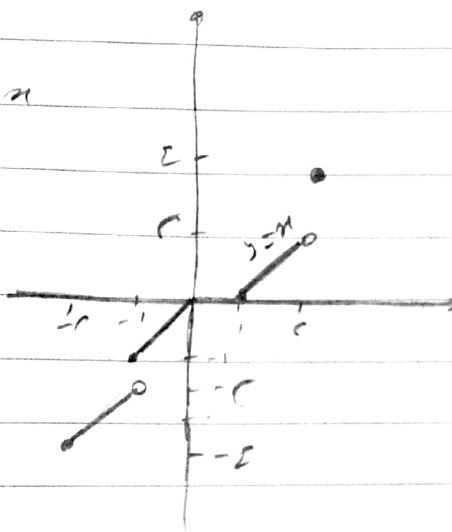
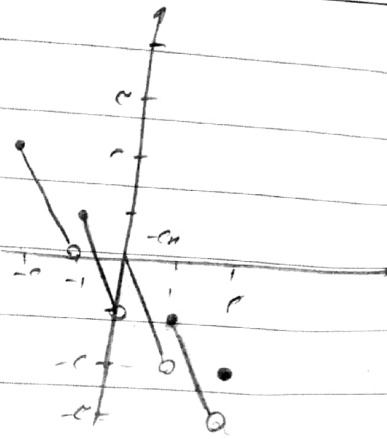
$1 \leq x < 2 \rightarrow y = 1$

$2 \leq x < 3 \rightarrow y = 2$

0

$-1 \leq x < 0 \rightarrow y = -1$

$-2 \leq x < -1 \rightarrow y = -2$



3) $y = \lfloor x \rfloor - \lfloor \pi x \rfloor$

$RP = [0, 1)$

1) $y = \lfloor x \rfloor - \lfloor \pi x \rfloor$

$RP = [0, 1)$

$\lfloor x \rfloor - \lfloor \pi x \rfloor$

$0 \leq x - \lfloor \pi x \rfloor < 1 \rightarrow \frac{x}{\pi} \leq \lfloor \pi x \rfloor < x$

2) $y = \lfloor \frac{x}{\pi} \rfloor \rightarrow y = \lfloor \frac{x}{\pi} - \lfloor \frac{x}{\pi} \rfloor \rfloor$

$0 \leq \frac{x}{\pi} - \lfloor \frac{x}{\pi} \rfloor < 1 \rightarrow \frac{x}{\pi} \leq \lfloor \frac{x}{\pi} \rfloor < \frac{x}{\pi} + 1$

$RP = [0, 1)$

3) $y = \lfloor \pi x \rfloor - \pi x$

$RP = (-1, 0]$

4) $y = \lfloor a \sin x \rfloor + \lfloor b \cos x \rfloor$

$-\sqrt{a^2 + b^2} \leq \lfloor a \sin x \rfloor + \lfloor b \cos x \rfloor \leq \sqrt{a^2 + b^2}$

$-\sqrt{a^2 + b^2} \leq a \sin x + b \cos x \leq \sqrt{a^2 + b^2}$

$RP = [-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}]$

REEF GROUP

حل المسألة

$$\rightarrow y = r \sin x - c \cos x$$

-4

$$c \sin x + (-c) \cos x$$

$$R_y = [-c, c]$$

$$\text{مثال } y = [r \sin x + c \cos x] \rightarrow R_y = [-\sqrt{r^2 + c^2}, \sqrt{r^2 + c^2}]$$

$$-\sqrt{r^2 + c^2} \leq r \sin x + c \cos x \leq \sqrt{r^2 + c^2} \rightarrow [-\sqrt{r^2 + c^2}, \sqrt{r^2 + c^2}]$$

$$\text{مثال } y = \sqrt{\cos x - 2 \sin x} = [0, \sqrt{2}]$$

$$[-\sqrt{2}, \sqrt{2}]$$

$$\text{مثال } y = 3 \sin^2 x + 4 \sin x + 1 \rightarrow \frac{1}{2} y \rightarrow \text{max} \rightarrow R_f = [0, 4]$$

$$\sin x \in [-1, 1] \rightarrow \frac{y}{2} = \frac{b}{a} = \frac{4}{2} = 2 \rightarrow y = 4$$

$$\rightarrow y = 2 \sin^2 x + 3 \sin x + 1 \rightarrow \sin x = 1 \rightarrow \text{max}$$

$$\sin x = -1 \rightarrow y = 1 \rightarrow \text{min}$$

$$R_f = [\frac{1}{2}, 4]$$

$$\sin x = -\frac{b}{2a} = -\frac{3}{4} \rightarrow 2 \left(\frac{9}{16} \right) - \frac{9}{2} + 1 = \frac{1}{2}$$

$$\text{مثال } y = 8 \sin^2 x + 2 \cos^2 x \rightarrow y = 7 \cos^2 x + 1 \rightarrow \text{min} \rightarrow 1$$

$$8 \sin^2 x + 2 \cos^2 x = 7 \cos^2 x + 1$$

$$\cos^2 x = \frac{1}{7} \rightarrow \cos x = \pm \frac{1}{\sqrt{7}}$$

$$R_f = [1, 8]$$

$$\text{مثال } y = \frac{r \cot x}{1 + \cot x} = \frac{r \cos x}{\sin x} = r \cos x \frac{\sin x}{\frac{1}{\sin x}} = \sin x$$

$$R_f = [-1, 1]$$

Übersicht

$$\text{ii) } y = \frac{\sin^4 n + \cos^4 n}{a} = \frac{1 - 2 \sin^2 n \cos^2 n}{a} \quad -9$$

$$0 < a < 1 \quad R_f = \left[\frac{1}{a}, 1 \right)$$

$$\Rightarrow y \in \left[\sin^4 n + \cos^4 n \right] = [0, 1]$$

$$\text{iii) } y = \frac{n^2 + n - 2}{n + 2} = \frac{(n-1)(n+2)}{(n+2)} = n-1 \quad -10$$

$$n \neq -2 \quad R_f = \left[\frac{-9}{3}, 3 \right)$$

$$\text{iv) } y = \frac{n^2 - 1}{n - 1} = \frac{(n-1)(n+1)}{(n-1)} = n+1 \quad n < 1$$

$$\text{ent } \left/ \frac{y}{n} = \frac{1}{n} \right.$$

$$R_f = \left(-\infty, \frac{1}{2} \right] - \{0\}$$

$$n \neq +1$$