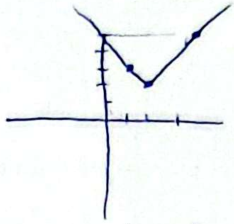
نام و نام خانوادگی: آیلین عظیمی — تکلیف ۷ — کلاس: پنجشنبه یازدهم ریاضی

۱

الف) $y = x + |2x-4|$: (2)

$x<2$	$x \ge 2$
$x-2x+4 = -x+4$	$x+2x-4 = 3x-4$
$f(1)=3$	$f(3)=5$

$f(2)=2$

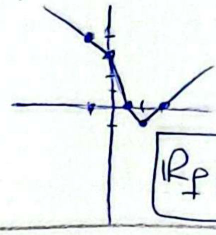


$R_f : [2, +\infty)$

ب) $y = |x-2| + |x-1| - |x|$:

$x<0$	$0 \le x<1$	$1 \le x<2$	$x \ge 2$
$-x+2-x+1+x = -x+3$	$-x+2-x+1-x = -3x+3$	$-x+2+x-1-x = -x+1$	$x-2+x-1-x = x-3$
$f(-1)=4$	$f(0)=3$	$f(1)=0$	$f(2)=-1$

$f(3)=0$



$R_f : [-1, +\infty)$

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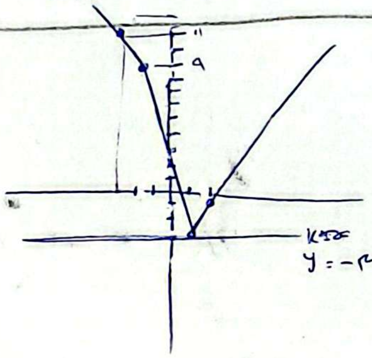
$|3x-3| - |x+2| = k$ (1) (-2)

$x<-2$	$-2 \le x<1$	$x \ge 1$
$-3x+3+x+2 = -2x+5$	$-3x+3-x-2 = -4x+1$	$3x-3-x-2 = 2x-5$
$f(-3)=11$	$f(-2)=9$	$f(1)=-3$

$f(2)=-1$



تنها خطی که در اثر برخورد با شکل ۱ ریشه می دهد خط $y=-3$ است

$k=-3$

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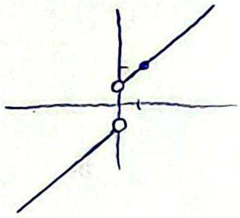
الف) $y = x + \frac{x}{|x|}$:

$x<0$	$x>0$
$x-1$	$x+1$
$f(-1)=-2$	$f(1)=2$

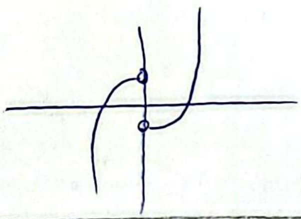
$f(0) = \times$، $x \ne 0$



ب) $y = x|x| - \frac{x}{|x|}$:

$x<0$	$x>0$
$-x^2+1$	x^2-1

$x \ne 0$



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الف) $y = [x] - 2x$ $\xrightarrow{-2 \le x \le 2}$

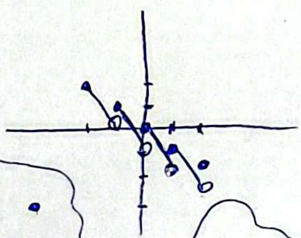
$-2 \le x < -1 \xrightarrow{[x]=-2} y = -2x-2$

$-1 \le x < 0 \xrightarrow{[x]=-1} y = -2x-1$

$0 \le x < 1 \xrightarrow{[x]=0} y = -2x$

$1 \le x < 2 \xrightarrow{[x]=1} y = -2x+1$

$x=2 \longrightarrow [x]=2 : y = -2x+2$



ب) $y = [x]|x|$ $\xrightarrow{-2 \le x \le 2}$

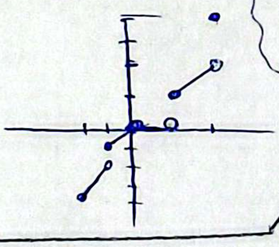
$-2 \le x < -1$; $(-2)(-x) = 2x$

$-1 \le x < 0$: $(-1)(-x) = x$

$0 \le x < 1$; $(0)(x) = 0$

$1 \le x < 2$; $(1)(x) = x$

$x=2$; $(2)(x) = 2x$



ج) $y = x - 5\left[\frac{x}{5}\right] = 5\left(\frac{x}{5} - \left[\frac{x}{5}\right]\right)$ $[0,1)$ $\rightarrow R_f : [0, 5)$

د) $y = [2x] - 2x \rightarrow 0 \le 2x - [2x] < 1$

$0 \ge [2x] - 2x > -1$ $\rightarrow R_f : (-1, 0]$

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الف) $y = 3x - [3x] \longrightarrow 0 \le a - [a] < 1 \rightarrow R_f : [0, 1)$

ب) $y = 4x - 4[x] = 4(x - [x]) \rightarrow 0 \le x - [x] < 1$

$0 \le 4(x - [x]) < 4 \longrightarrow R_f : [0, 4)$

ب) $y = 3\sin x + 2\cos x$ $\quad [-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}]$ $\quad -\sqrt{13} \le 3\sin x + 2\cos x \le \sqrt{9+4}$ $\quad \sqrt{13}$ $\Rightarrow$ $R_f : [-\sqrt{13}, \sqrt{13}]$

ج) $y = 4\sin x - 3\cos x \rightarrow -5 \le 4\sin x - 3\cos x \le \sqrt{16+9}$ $\Rightarrow$ $R_f : [-5, 5]$

د) $y = [2\sin x + 5\cos x] \rightarrow -\sqrt{29} \le 2\sin x + 5\cos x \le \sqrt{4+25}$ $\Rightarrow$ $\left[[-\sqrt{29}, \sqrt{29}]\right]$: $R_f : \{y \mid y \in \mathbb{Z}, -5 \le y \le 5\}$

ه) $y = \sqrt{\cos x - 2\sin x} \rightarrow -\sqrt{5} \le \cos x - 2\sin x \le \sqrt{1+4} \rightarrow \sqrt{[-\sqrt{5}, \sqrt{5}]}$ $\Rightarrow$ $R_f : [0, \sqrt[4]{5}]$

الف) $y = \sin^2 x + 3\sin x + 2 = (\sin x + 1)(\sin x + 2)$

$\max \sin x : -1 = (-1)(-1) = 1$

$\min \sin x : 1 = (1)(2) = 2$

$\Rightarrow$ $R_f : [0, 2]$

ب) $y = 2\sin^2 x + 3\sin x + 2$

$\max \sin x : -1 = 2 - 3 + 2 = 1$

$\min \sin x : 1 = 2 + 3 + 2 = 7$

$\Rightarrow$ $R_f : [\frac{1}{8}, 7]$

ج) $y = \sin^2 x + 4\cos^2 x = \sin^2 x + 4 - 4\sin^2 x = -3\sin^2 x + 4$ $\quad [0,1]$ $\quad [-3, 0]$

$-3 + 4 = 1$, $4 + 0 = 4$ $\Rightarrow$ $R_f : [1, 4]$

د) $y = \frac{2\cot x}{1 + \cot^2 x} = \frac{\frac{2\cos x}{\sin x}}{\frac{1}{\sin^2 x}} = 2\sin x \cos x = \sin 2x$ $\quad [-1, 1]$ $\rightarrow$ $R_f : [-1, 1]$

الف) $y = \sin^4 x + \cos^4 x = 1 - 2\sin^2 x \cos^2 x = 1 - 2\sin^2 x + 2\sin^4 x = 2(\sin^2 x)^2 - 2\sin^2 x + 1$

$\sin^2 x$	min: 0	max: 1	$\frac{-b}{2a} = \frac{2}{4} = \frac{1}{2}$
	1	1	$\frac{1}{2} - \frac{2}{4} + 1 = \frac{1}{2}$

$\Rightarrow$ $R_f : [\frac{1}{2}, 1]$

ب) $y = [\sin^8 x + \cos^8 x] \rightarrow$

$1 \ge \sin^{2k} x + \cos^{2k} x \ge 2^{1-k}$

$1 \ge \sin^8 x + \cos^8 x \ge 2^{-3}$

$\left[\left[\frac{1}{8}, 1\right]\right] \Rightarrow R_f : [0, 1]$

(۱۵)

الف) $y = \dfrac{۲x^۲ + ۷x - ۴}{x + ۴} = $ $\rightsquigarrow x^۲ + ۷x - ۸ = (x + \frac{۸}{۲})(x - ۱) = \dfrac{(x + ۴)(۲x - ۱)}{(x + ۴)} = ۲x - ۱$

$x \neq -۴$

$y \neq ۲(-۴) - ۱ \neq -۹$

$\begin{cases} R_f : R \\ y \neq -۹ \end{cases}$

$R_f : R - \{-۹\}$

$x \neq$

د) $y = \frac{x^3 - 1}{x - 1} = \frac{(x-1)(x^2 + x + 1)}{(x-1)} = x^2 + x + 1$

$-\frac{\Delta}{4a} = \frac{-(1-4)}{4} = \frac{3}{4} \rightarrow$

$R_f : [\frac{3}{4}, +\infty)$